The Arithmetic Teacher

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A Journal of
THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

THE ARITHMETIC TEACHER

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The National Council of Teachers of Mathematics

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THE ARITHMETIC TEACHER

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"To Hold, as't Were the Mirror up to Nature; to Show the Very Age and Body of the Time"*

RUDOLPH E. LANGER
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THE DAYS ARE NOT LONG PAST since mathematical instruction, beyond a very elementary stage, was decried by many an ostensible educational leader as having little value except for a small minority of the school population. To that judgment we opposed ourselves, stubbornly, but not always with success. We were heard with forbearance, as one does the defenders of a vested interest, but the place of mathematics in the general curriculum was nevertheless minimized. Other subjects, less intellectually burdened, but allegedly more immediately relevant to daily living, were put forward as more appropriate and important.

Time has hastened to disconcert those who carried this issue against us. Today others, without any stimulus on our part, are calling for more widespread mathematical competency, and are doing so with an insistency that astonishes even us. They are telling the public—no, they are warning it—that a great increase in the supply of persons with mathematical training is nothing less than critical for our national safety, for the very preservation of our mode of life. Is this an over-correction? Is the pendulum which was formerly pushed too far to one side now swinging too far in its return? I thought it might suit this occasion not badly for us to consider this matter, or, in the words of Shakespeare: "To hold, as't were, the mirror up to nature; to show the very age and body of the time."

Many a river, as it flows along its course from source to sea, passes in turn through reaches of calm and ones of turbulency. Here it glides with a deep and easy sweep, only to plunge there into confusion, to tumble and rush chaotically without direction but with much noise toward no apparent goal. The unfolding of history in the stream of time seems sometimes to be of a similar pattern. Quiescent eras, placid and peaceful, give way, as it seems sometimes precipitately and unaccountably, to troublous ones, in which the orders of the day are tensions and conflicts. We find ourselves now in such a disturbed era. Diplomatic and economic crises succeed each other in dismaying tempo, and military ones constantly impend. Our destiny is clouded. Anxiety overshadows us, as we contemplate terrors of war and threats to freedom. Our continued liberty to choose, without hated outside dictation, what shall be the cultural patterns and practices to which we shall subscribe, seems suddenly to be in jeopardy. Certainly the historical narrative, as it will be told in the future, will not pass our times by as eventless and dull.

* Address delivered to National Council of Teachers of Mathematics at Ann Arbor, August 18, 1959.

Some of us can remember a calmer past. But when we do so we recall also that the mathematics teacher was not then exalted as he or she now is. Mathematics, at least as a curricular subject, was then discounted. It was conceded to have worth only for a few. In the curriculum it was being progressively dislodged from the center, even threatened with complete eclipse. The yardstick of much educational doctrine was then immediate practical utility. Subjects free from baffling difficulties and from the importunities of logic were preferred—as it was said, to protect the pupil's egos from harmful experiences of frustration. The "hard" subjects, mathematics prominently among them were diluted, softened up, minimized or displaced.

That such was deluded doctrine, time has hastened to show. The realities of life are not to be conjured away by wishful thinking, nor does the race go to the soft and untrained runner, however much unwarranted self-confidence may have been instilled into him. Now the land rings with calls for sturdy talent, for men and women trained, to whatever stage, but thoroughly, in "hard" mathematics. You, as mathematics teachers, have become objects of national concern, and have been projected into a completely central educational role. What are the reasons for this transfiguration?

The most distinctive traits of our culture —the culture of the West—at least as it has evolved in its later course, have been a devotion to science and exploitation of science in technology. The origins of these activities lie far in the past. Their dominance, as social forces however, dates back only into the last century. In the more recent past, they have burst forth to sheerly explosive proportions, so that presently they effectively shape and direct our lives. Now mathematics is the mainstay of both science and technology. For it is the medium by which the human genius records its observations upon the structure and behavior of nature, and from which it draws the power to discern the underlying order. It supplies man, in short, with the basis for broadening his understanding and for bending his knowledge to useful ends.

This whole matter of the character of mathematics and its role as a human enterprise is, unfortunately, all too little understood, despite the fact that in a literate society such as ours everyone is at some time exposed to some mathematical instruction. The great majority of people, even of those who are otherwise well informed, are utterly unaware of both the vastness and the diversity of mathematical thought, and of its living dynamic character. In no measure whatever do they realize that mathematics is the central directive agent of the whole human quest for quantitative understanding of the environment, that it is in command at all points in man's efforts to apply nature to his advantage, that it is the medium through which chaos and superstition are banished, and that it is the backbone not of one science or of several, but of all science.

The Role of Mathematics

A scientific doctrine comes to stature and is accorded authority only where, and only to the extent that, it is formulated in mathematical theory. The invention of applicable theories, and their enlargement and perfection, are the mathematician's domain. Only where appropriate theories are at hand, are the roads to scientific advance and technological development open. The assignment to keep the roads open, is a continual spur upon mathematics to extend its capability through growth and diversification. Scientific evolution would soon cease in the absence of mathematical expansion. The great technological achievements of our century —transportation by flight through the atmosphere or under the surface of the sea, the mass communication mediums of radio and television, the recording of music, atomic fission and fusion, radar detection and rocket power, the analysis of spatial radiations, the expanded knowledge of the astronomical universe and of the constitution of the earth—all these have, in their appropriate measures, been mathematically achieved.

The inner growth of mathematics, which is thus positively essential, is, fortunately, in fact going on. It is even vigorous, although, to be sure, it is not often popularly proclaimed like many more easily apprehended scientific or technological triumphs. New mathematical theories continue to be born, and as lusty infants receive widespread attention, while older ones, even venerable ones, are restlessly being rejuvenated and revivified on the basis of new ideas. A single instance of this, from the very immediate past and even from the present, is the development of the high-speed electronic computting machines. These machines are nothing other than mathematical logical structures which engineers have incorporated into hardware. With them mathematics has magnified its own powers so immeasurably in many fields that it can now deal with whole categories of problems that were sheerly beyond its, beyond any human, capability only a score of years ago.

The belief has been voiced that this era in which we now find ourselves will be known in the future as the *technological revolution*, and that its effects will be no less far-reaching and profoundly reforming than were those of the industrial revolution of the eighteenth and nineteenth centuries. It is certainly true that industry is in a revolutionary state. Its production rates, and the variety of its commodities, especially of such as were only recently discovered or invented, are unprecedentedly immense, whereas technology has so heightened production efficiencies that increasing tasks are accomplished with ever fewer hours of labor. Routines are being transferred more and more from human hands to machines. In consequence of that employment is being shifted in large measure from persons with less to ones with more technical training, by which is often meant to ones with more knowledge and skill in mathematics.

International Implications

These changes, even were they confined to our own Country, would be deeply affecting. They are, however, not so confined, and by that very fact their impacts are ominously magnified. They have destroyed the bases of former international alignments. New destinies have been born of them, and make themselves felt in pressures and clashes of interests that are burdened with the dangers of war. In the new order which has thus come about, the rise of Russia to the position of a world power is conspicuous. It is a direct result of industrialization. To recognize that, is inevitably to recognize also that power among nations will be held by the technologically strongest. In the past the dominant power has been ours. To wrest that from us the Soviets have thrown themselves into the struggle for supremacy in the technological field with a singleness of purpose and an intensity of effort which considerably surpass our own.

The Soviets give their allegiance to political tenets and social ideologies that are poles apart from ours. Their opposition to us is therefore disquieting, and loaded with threats to many of our material interests. And not only to those, but more profoundly to the continuance of our freedom to fashion and direct our own social, economical, political and religious institutions in the ways we prefer, without alien dictation, and to wield and enjoy justice as we conceive it. With what seems like desperate suddenness we find ourselves in jeopardy of all the devastation that man can loose by his mastery of natural phenomena. For technology, when turned to destruction, operates with terrible effect. We are pitted against an adversary who is swelled with arrogance in a new-born awareness of power. Our doctrines are hateful to him, or at least to his coterie of leaders, and he sees in us the agency which, and which alone, threatens the frustration of his gratuitous ambitions. The engagement in which we are thus locked is an unsparing one, in which anything less than our best total effort is certain to be disastrously insufficient.

The Soviets have first-rate scientists and mathematicians. They are resolutely working to increase greatly their supply of trained personnel. The leadership which

was ours is slipping away; the prospect of losing it wholly confronts us. In point of natural resources we are matched, in sheer numbers of men we are completely outmatched. That being so, our hope to maintain an ascendant position must be primed to the shrewdest possible use of our wits, that only means by which one can magnify oneself to compete with a physical superior. This is what the mirror shows when we hold it up to nature. This is the age and body of the time.

The fact to which we have thus been rudely awakened, is that technological progress, that beneficent genie that has heretofore given us our high standard of living, cannot henceforth be safely left to its own recourse, but has become one whose favor we must sue to the utmost and whose welfare it is imperative for us to hasten. Upon that our future depends, and that, therefore, is a matter that impinges quite directly upon all of us in this room. For the future of a Country is shaped, in the very first instance, by its educational effort, which is to say by its schools. The burden of a renewed and heightened effort therefore falls upon them. And because their strategic importance is clear, we suddenly find the schools admonished of the ominously developing national need in the pronouncements of committees, commissions and councils, and by advisors to the highest authority of the land. Public attention is being directed upon the schools. Teachers and pupils have become subjects of discussion in circles which in other times rarely gave them a thought. Reappraisals of the schools' responsibilities to the social body are being called for, and the demand is heard for a reorientation of the schools to tougher and more realistic objectives. The need for more teachers is stressed, and the inequities with which teachers have had to content themselves in the past are being acknowledged and promised of redress.

Among the challenges of the day none is being sounded more insistently than the call for more, and more effective teaching of mathematics. From that, every mathematics teacher surely can draw a measure of gratification, for it manifestly emphasizes the importance of the mathematics teacher's job. Personal and professional satisfaction root upon that, but so also does larger responsibility. In countries that are under the sway of dictatorial rule, pupil enrollments in mathematics classes, or, for that matter, in any classes, can be raised by edict and maintained by regimentation. Those expedients are not ours; we neither can nor wish to use them, since we have no remote desire to abrogate each individual's right to choose in matters of his own fortune. Persuasion and guidance are less prompt than compulsion, and are more demanding of patience and devotion. Nevertheless they seem to us to be the preferable means, and to be more certain of honest results.

With pupils exposed in these days to repeated emphasis upon the challenging roles and dynamic character of science and technology, through almost uninterrupted broadcasts of news and commentary upon current events and trends, the task of persuading them of the importance of mathematics for many an exciting career should not prove difficult. The main problem may be to correct the belief that such careers are only for super-men. The facts are that opportunities, for both men and women with mathematical training, be that through the advanced or the intermediate stage, or even only through the elementary stage, are now numerous, varied, and well rewarded, as they never were before. Careers are open in industry and in many branches of the Government, and, of course, also in the schools themselves. Where positions calling for mathematical training were few in the not very distant past, there are now many, and it is to be foreseen that even a greatly increased supply of trained persons will continue to fall short of the growing demand. In our whole economy the trend is from less trained to better trained personnel.

Mathematics in the Schools

The call for the more effective teaching of mathematics is being met by efforts, many of them presently under way, to reform the

curriculum, on the one hand, and to bolster teacher competency on the other. As to the curriculum, the attention is being directed upon a revaluation of subject matter, with an eye toward minimizing the more dispensable items to permit the introduction of ones that seem to be more relevant to modern needs. The importance of this cannot be questioned. I shall refrain from discussing it, however, both for the reason that I have not personally had a hand in it, and because I know that many of your deliberations at this meeting center upon it.

The awakened recognition of the importance of mathematics has, not unnaturally, called forth a demand for higher competencies on the part of its teachers. As a social force mathematics has changed the times. But it has also changed with the times, and is indeed continually changing. Even in the very recent past fruitful new ideas in considerable number have been conceived by it, and on the basis of these new disciplines have been born and older ones reconstructed. While much of this is recondite and specialized, some of it is, on the contrary, easily accessible and well adapted to enrich teaching backgrounds. The appreciation of this has led to the proffering of teacher institutes, many of them during the summer months, by a number of educationally oriented agencies. These afford teachers the means both to refresh and extend the stocks of their professional assets. Profit from them will depend, of course, upon the teachers' response. Professional competency is not something that can be embalmed to maintain itself. If it is not continually refreshed, it decays. You, who have had the enterprise to attend this national meeting, are not those who need exhortation upon this point. You are, however, only a small minority of all mathematics teachers, whereas the serious demands of the times will impinge upon all alike.

With the advance of industrialization, the general welfare of society has come to depend more and more upon human acuity and wit. Leadership of high intellectual calibre has become more and more essential in all enterprise, whereas labor and brawn have become more dispensable. Finding ourselves locked, as we now presently do, in an international engagement which foreseeably will strain our every capability to the utmost, and in which failure has a most malevolent aspect, we are faced with the fact that leadership talent is the critical asset above all. The detection, the development, and the proper employment of such talent, with the greatest possible promptness and efficiency now confronts us as an imperative.

The call is accordingly going out—again to the schools—to be alert to the early singling out of pupils of superior abilities, and to open ways for their optimal development. The circumstances which are spurring this prod upon the schools are regrettable, but the prod itself, I am far from regarding as such. For it restores to a better perspective a matter of which, it has long seemed to me, the schools have not been duly mindful. Equality of educational opportunity for all is a tenet we have enshrined; one to which we will continue to cling. It is mistaken, however, to read into that tenet an implication that pupils must all be treated alike regardless of their unequal capacities to profit therefrom. The total endeavor of the schools is limited by the material and human resources available to them. The maximum return they can give—in their role as an organ for the total social good—may, therefore, be expected to depend upon a strategic allocation of effort. To consider the keeping-up of the rear guard as the matter of first importance, as a matter to be pressed even at the expense of appropriate attention to the whole column's advance, seems hardly an admirable strategy. Yet it is one to which allegiance has all too widely been given. The schools are being urged now, with considerable insistence, to look more to the head of the column, and when the occasion is ripe, to send out advance guards ahead.

This is not the first time that I have fallen upon this theme before this National Council. At your summer meeting at Denver, Colorado—I believe it was just ten years ago—I had the honor, as I have it now, to

be your banquet speaker. The burden of my message then bore especially upon this matter of the superior student. Some observations I made then are still germane. I will repeat them in the identical words. "It is a fact of nature that in any society the genius for leadership resides in only a small minority of the people. But the whole impetus toward advance, the whole implementation of progress, comes from this few. In statesmanship, in science, in education, in the arts and in business the leaders are the exceptions. The great majority of any population plays no role in shaping its own destiny, but depends inertly upon the leaders, who are the brain of the social body. The best that education can do for the many average pupils would come to little were they not to receive guidance later from the superior minds."

"In recent years we have been drawn more than ever before into international relationships. Our rivals are intelligent energetic peoples, who are resourceful and determined—not to be safely underestimated. They do not see eye to eye with us. That being so, the most elementary considerations must tell us that our best efforts will be needed to hold our own."

"We are pitted against people who have a lively reverence for leadership. In their educational effort they are giving this their foremost attention. Is it the part of wisdom for us to continue our present neglect of our better students?"

The ten years since I wrote those words have subtracted nothing from their truth, but have added greatly to their urgency. The national leadership, which was then ours, has in large measure slipped from our own grasp, and has fallen into almost precarious balance. The way on how the beam shall tilt in the future will rest largely with the schools.

The profession of teacher is an exalted one. If life under our institutions and by our standards has values—as I am sure we do believe—those values are summed up in our understanding of the universe, our reverence for its orderly scheme, our faiths, our tenets of conduct and justice, our dedication to freedom for initiative and expressive creation, and our respect for the dignity of the human individual. The orderly transmission of this wealth from the passing generation to the oncoming one, and the inculcation of such respect and love for it as will insure its defense and preservation, that is the teacher's mission. If we hold the mirror up to nature, it shows us that the age and body of the time is one in which our prospects are darkened and our own heritage threatened. Teachers—mathematics teachers—if importance of the social niche in which your role is cast inspires you—and who among us is not thereby inspired—yours is a job that is worthy of magnificent effort.

EDITOR'S NOTE. This is indeed an interesting and perhaps troublesome period of history. Professor Langer has very beautifully described the role of mathematics in the advances of the era. Subscribers to this journal might well pass this article on to others to read so that more may be informed concerning mathematics and its role in our society. The teacher of mathematics at any level has a serious responsibility not only to the large number of typical students but particularly to those few who have the ability to pioneer in the frontiers of mathematical thought. Frequently the spark of genius can be identified in the elementary school and here is the spot to insert the type of encouragement and stimulation that will stir such a pupil to inquire, discover, and proceed perhaps independently. If a teacher is lacking in the insight and in the mathematical knowledge which are needed to enhance the work of the "gifted pupil" the opportunity may be lost. But such a teacher should feel free to seek a source of help instead of forcing the individual to become a conformist to the basic program set for the average. We must hold high the national welfare of the United States of America and aid those who may become the pioneers in intellectual pursuits. Mathematics is one of these pursuits and many of us believe it to be the most basic.

The Hundred-Board

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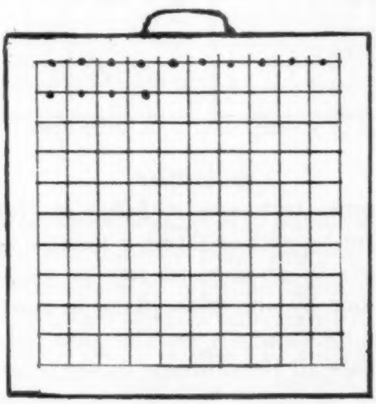
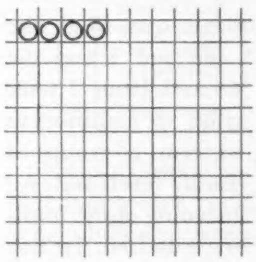
EVERY TEACHER OF ARITHMETIC should have a *hundred-board.* The writer believes it is one of the most efficient teaching aids for use in the development of understandings of concepts and operations with number quantities. If a teacher could select but one gadget for use in the arithmetic classroom he should choose the hundred-board because of its many uses with regard to both the scope and grade placement of material.

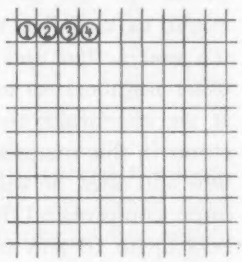
The hundred-board can take one of several forms. The simplest home-made board consists of a square of one-half inch plywood large enough to be ruled off into 100 smaller squares each 2 inches by 2 inches, with a 2 inch border around the outside. A finishing nail should be driven at the center of the top line of each small square. The finished product should then be stained and equipped with a handle to facilitate handling, hanging, and storing. In this form it can be used on a table, stand, easel, tripod, or in a chalk tray.



Variations of the above can be effected by use of pegs or hooks placed at the exact center of each small square, by increasing its size so that it can be set on the floor for easy handling by small children, or can be constructed with feet so that it can be used while resting in a horizontal position on a desk or table.

The teacher should have a set of numbers from 1 to 100 and a set of one hundred unnumbered objects which can be hung on the board. The numbers can be purchased commercially or prepared by numbering oaktag labels, department store price tags, or poker chips punched for hanging. The teacher should use objects which permit easy handling and which at the same time are attractive, such as spools, curtain rings, rubber washers, identification tags, one-inch pieces of plastic hose, and similar objects.

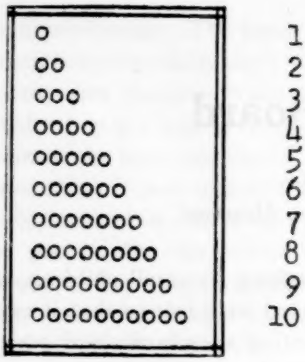




In the paragraphs which follow the writer will show a few uses of the hundred-board.

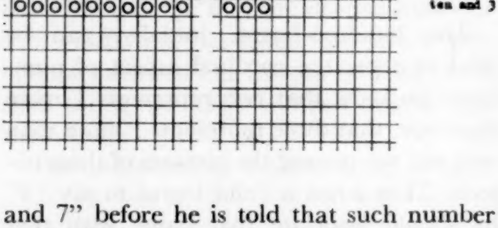
Number

The hundred-board, initially, can be used to show the cardinal aspect of numbers. To show that two represents 1 more than one, that three represents 1 more than two, etc. we present the pictures of these objects. Thus when a child learns to say "4" he should associate that name with that things. This is "fourness." The meaning of the number expressions from one to ten should be shown on the board as follows:



Now after a child has learned the meaning of "ten" he can count tens. Reference to a hundred-board filled with numbers clearly reveals groups of tens and the child should be taught to count them as tens. It should be as easy for him to comprehend 1 ten, 2 tens, 3 tens, 4 tens, etc. as it is for him to learn to count 1 marble, 2 marbles, 3 marbles, . . . or 1 case, 2 cases, 3 cases, etc.

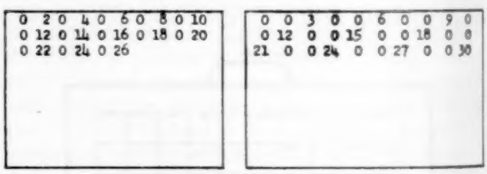
Next we introduce the "teen" numbers with the help of a second hundred-board. Each row of the hundred-board represents a ten, so if a filled hundred-board is placed beside the one pictured above the child ought to visualize number quantities like ten and 1, ten and 2, ten and 3 and the other numbers up to ten and 10 or 2 tens. If we encourage the child to continue to count in this manner, in terms of groups of tens and ones, he will readily learn and, we hope, comprehend the meaning of the numbers up to 10 tens or one hundred. In this way the child will learn to say expressions like "3 tens and 5," "3 tens and 6," "3 tens



and 7" before he is told that such number expressions can be shortened to thirty-five (35), thirty-six (36), and thirty-seven (37). When the child knows that four full rows of discs represent 4 groups of ten discs he can be shown how to express this quantity symbolically as 40. An extension of this concept should enable him to replace all the discs with others numbered to 100.

We can use the numbers on the hundred-board for teaching the child to count by twos. We begin by having all of the numbers turned over (face down). We ask the child to skip the first disc, number 1, and turn over the next disc. We skip one again and turn over the next one, we skip one again and "face-up" another. If we continue doing this until we reach 50 or 60 the child should observe the pattern of endings and should observe that full columns are turned down and full columns are exposed. Those numbers exposed are called "even" numbers and all of them end in either 2, 4, 6, 8, or 0.

The same procedure might be followed when teaching children to count by threes, fours, or fives. If all of the discs are turned down, then we skip two and turn over one, skip two and turn over one, etc. when counting by threes. The numbers exposed will be 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, etc. The pattern of endings does not begin to repeat until after we have shown ten threes.



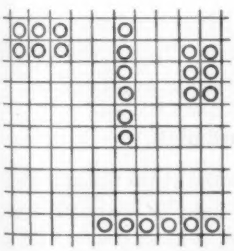
Another technique to follow in learning to count by twos or threes is to stack all the numbers in an ordered row or pile, take them one by one, place them in rows of 2 or 3 as the case may be. One does not have to replace all 100 numbers on the board but enough to enable him to understand the concept of counting by groups and to learn the respective patterns. The number board then will look like this:

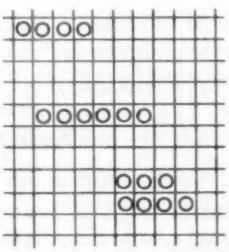
1	2	21	22
3	4	23	24
5	6	25	26
7	8		
9	10	etc.	
11	12		
13	14		
15	16		
17	18		
19	20		

1	2	3	31	32	33
4	5	6	34	35	36
7	8	9	37	38	39
10	11	12			
13	14	15		etc.	
16	17	18			
19	20	21			
22	23	24			
25	26	27			
28	29	30			

The hundred-board is useful for picturing the various patterns of number quantities. The quantity 6, for instance, may be shown on the number board in several patterns. If children re-arrange discs (a constant number of discs) they will discover numerous facts concerning the given number. This activity develops insight into number quantities and number relationships.

Groups of different sizes may be pictured on the board and children challenged to determine the largest group. If they see groups containing 4, 6, and 7 spools they ought to be able to deduce that 6 contains (2) more than 4, that 7 contains (1) more than 6, and that 7 contains (3) more than 4. Conversely they will learn that 4 and 6 are smaller than 7 by 3 and by 1 respectively. The comparison phase of counting can be taught through the use of the hundred-board.


Groups of 6


Groups of 4, 6, and 7

Addition

The hundred-board is very useful in helping children discover basic addition facts. Since addition is a "putting together" operation we want to know how many we have altogether if we put 3 discs with a group of 4 discs. We show 4 discs on the top row and 3 discs on the second row and then combine them into one group. We discover that when we put 1 (of the 3) with 4 we will have 5, another 1 will make 6, and the last 1 will make 7. Thus 4 discs and 3 discs are 7 discs. Children will discover that there are many addition facts whose sums are less than 10—the combining of two small sets of discs will often leave the top row incomplete. Also, in learning the meanings of the numbers 1 to 10 they will have discovered how many more are needed to complete the row of 10. For instance, the last row of discs needs 0, the next to last row needs 1 so $9+1=10$, the row containing 8 needs 2 so $8+2=10$, etc. In working with the complete set of 10, children ought to learn the sub-sets which add to 10. The board showing the sum of 4 discs and 3 discs should gradually have the following appearance:

0000	becomes	00000	then	000000	and then	0000000.
000		00		0		

Manipulations of this type suggest to the students that addition represents an increase, a movement to the right—a forward motion.

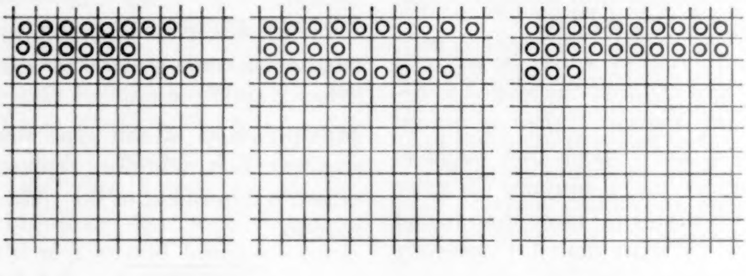
oxxxxxxxxx	1 + 9 = 10
ooxxxxxxxx	2 + 8 = 10
oooxxxxxxx	3 + 7 = 10
ooooxxxxxx	4 + 6 = 10
oooooxxxxx	5 + 5 = 10
ooooooxxxx	6 + 4 = 10
oooooooxxx	7 + 3 = 10
ooooooooxx	8 + 2 = 10
ooooooooox	9 + 1 = 10
oooooooooo	10 + 0 = 10

When children have mastered the combinations which total 10 they should have little difficulty recognizing sums which total more than 10. Since $7+3=10$, then $7+4$, $7+5$, $7+6$, etc. must be greater than 10. Let us show the addition of 7 discs and 5 discs on the hundred-board. The first row should show 7 discs. The second row should show 5 discs. The discs from the set of 5 should be moved one by one until we have completed the first row. We need 3 of these, therefore there should be 2 left on the second row. Thus the sum of 7 discs and 5 discs is the same as the sum of 10 discs and 2 discs. Since 10 and 2 represent 12, then the sum of 7 discs and 5 discs is 12 discs.

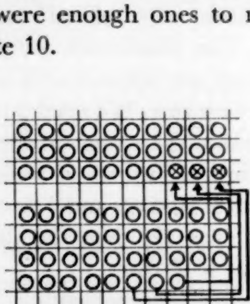
In adding $6+7$ (representing spools) children will visualize that they must take 4

of the 7 to put with the 6 to make a 10—therefore 6+7 is the same as 10 and 3. Addition with the hundred-board is similar to addition performed along a number line; the hundred-board is a number line decomposed into ten sections of ten where the concept of bridging is equally prominent. To add 6 to 7 on the number line one begins at the point marked 6 and moves forward (to the right) 4 places and then 3 more arriving at 13. Thus $6+7=13$.

When showing the addition of 3 or more quantities on the hundred-board children ought to sense the expediency of filling up groups of 10 so as to state the final sum as groups of tens plus ones. Thus $8+6+9$ when combined on the hundred-board becomes first $10+4$ (2 discs have been moved from the group of 6 to the 8 group making a complete 10). Then 6 of the 9 are moved to the second row completing another 10. Since there are 3 left on the third row the sum of $8+6+9$ is 2 tens plus 3 or 23. A student might move 2 of the 9 into the top row and 4 of the 9 into the second row to obtain the same result.



The rationale of carrying is evident when one demonstrates the addition of 2 or more two-digit numbers with the aid of the hundred-board. To show the addition of 27 and 38 we set up 2 rows of tens and 7 ones and then 3 rows of tens and 8 ones. To find the composite sum we use some of the 8 ones to fill in the row containing the 7 ones. Since 3 are needed to complete the row the final result shows 2 groups of ten, and 3 groups of ten (which we had in the beginning), and 1 more group of ten, and 5 ones. The sum, clearly visible on the hundred-board is 65. There were enough ones to make another complete 10.



$$\begin{array}{r} 27 \\ +\,38 \\ \hline 65 \end{array}$$

Two boards can be used effectively showing the tens on one board and the ones on the other. Then some of the ones can be used to make a ten. This procedure will enable children to "discover" that 2 tens and 7 plus 3 tens and 8 is the same as 6 tens and 5.

Subtraction

Subtraction is an operation which separates a given group into 2 or more subgroups and is usually taught first as a "take-away" operation, the opposite of the addition operation. If we wish to find the number of discs remaining after we have taken 3 discs from a group of 9 discs we first picture 9 discs on the hundred-board. When 3 of these are removed, one at a time, students should be encouraged to do so from right to left removing first the ninth one, then the eighth one, and then the seventh one. This backward operation will suggest a movement to the left along a number track . . . and that's the basic idea of "take-away subtraction."

Should we want to know the remainder when 5 things are taken from a group of 12 things we picture first the 12 things on the board. Children will see that the taking away operation (the 5 steps backward) will result in a remainder which is less than 10 They will first remove the 2 things on the second row and then move into the top row and take some from the group of 10. $12-5=?$

The same reasoning is applicable when we perform compound subtraction. To illustrate further let us subtract 8 from 23. First we fill in 2 complete rows and place 3 items on the third row to make 23. We start removing 8 of these one-by-one—we take off the 3 and then move into the group of 10 to get 5 more $(3+5=8)$. There will be only 15 left.

The solution of more difficult compound subtraction examples involving two digits illustrates the principle of reading and thinking from "left to right." Should we want to show the example

$$\begin{array}{r} 72 \\ -35 \\ \hline \end{array}$$

we would first show 72 items on the hundred-board. From these we are to remove 35. We will first take off 3 groups of 10 and then will take 5 ones. We take off the 2 single items and then take 3 more (enough to make 5) from one of the groups of 10. The final result is 37 and the algorism which will illustrate the thought pattern is as follows:

72	10	10	10	10	10	10	10 + 2
−35	10	10	10				− 5
				10	10	10	7

Multiplication

If the addends of an addition example are equal then the example can be solved by the operation called multiplication. Should we wish to add three groups of 2 we picture them as follows on the hundred-board:

× ×
× ×
× ×

We discover that when we regroup these items they will fill up the first 6 places. Thus 3 twos equal 6. These can also be shown with numbers after the result has been obtained—the numbered discs can be placed two on each row so that the numbers show 1 and 2, 3 and 4, and 5 and 6 on the three rows.

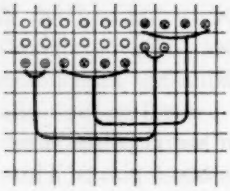
1 2
3 4
5 6

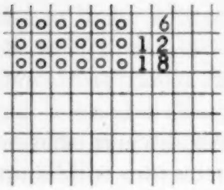
To show that 4 threes are 12 we arrange four rows of three discs and then regroup them to form sets of 10, if possible. There will be one set of 10 with 2 remaining. Thus $4\times3=12$. When numbered discs are used to show the representation of 4 threes the numbers will form the following pattern:

1 2 3
4 5 6
7 8 9
10 11 12

Children should become aware of the fact that the numbers in the last column are the "threes' facts" representing the products in the table of threes.

To show that 3 sixes are 18 we first put 6 discs on each of the first 3 rows. Now we regroup them to form sets of 10. In doing this we discover that there will be one set of 10 with 8 remaining. Thus $3\times6=18$. When the single discs are moved to fill in rows there are many ways in which this can be done—there is no particular "best" procedure. Again, when these discs are shown as numbered discs the numbers in the last column represent the products when 6 is the multiplicand. $1\times6=6$, $2\times6=12$, $3\times6=18$, etc.



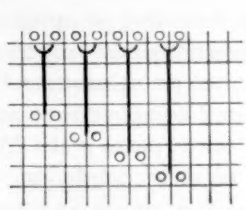


A hundred-board, with the addition of a few duplicate numbers, can be used to develop and display all of the multiplication facts and at the same time provide the opportunity for children to learn how to read tables. To do this, leave the first block of the hundred-board empty and hang all the numbers from 1 to 9 in the first row (9 will be in the place usually occupied by 10). Also, still leaving the first block empty, hang numbers from 1 to 9 in the first column. Now in the cells formed by the rows and columns hang numbers which represent the product of the two numbers which head the row and column. Thus in the cell of the row headed 4 and the column headed 7 put the number 28. Likewise in the cell of the row headed 7 and the column headed 4 put another 28. No other 28s are needed in the table. However, there are some numbers which appear more than twice as products within the table, namely, 8, 16, and 24.

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4							28		
5									
6									
7				28					
8									
9									

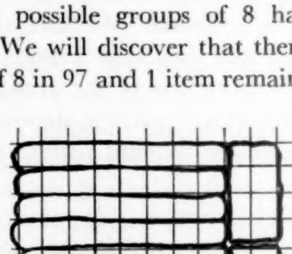
Division

Inasmuch as we have already shown ways to solve subtraction examples on the hundred-board, then division, which is also a separating or subtracting operation can also be illustrated thereon. There are two concepts of the division process, the measurement concept and the part-taking concept. We will first illustrate the measurement concept which asks one to find the number of groups when we know the size of the group (the multiplicand). An illustration of this idea is the question which asks how many groups of 2 can be formed from a group of 8. We begin with the dividend 8 and show that many things on the board. Now we remove 2 of these things and place them in a group elsewhere on the board. Then we repeat the operation with 2 more, and again 2 more until they have all been regrouped into sets of 2. We have discovered that 4 groups of 2 can be formed from a group of 8.



The example $16 \div 3$ asks us to find the number of groups of 3 objects which can be formed from a group of 16 objects. We illustrate 16 objects on the board and then pick them off in sets of 3. We discover 5 sets of 3 objects with 1 object left over or remaining.

The example $97 \div 8$ asks us to find the number of groups of 8 which can be formed from 97. Since the size of the sub-set is quite large and would require much tedious handling of objects we suggest the use of rubber bands, string, ribbons, etc. to encircle the group. We circle 8, then circle 8 more, etc. until all possible groups of 8 have been formed. We will discover that there are 12 groups of 8 in 97 and 1 item remaining.



The partitioning aspect of division asks us to find the size of each sub-group when the number of groups is known. For instance, $6 \div 3$ might ask us to find the number of

items in each group if 6 items are divided into 3 equal groups. We show 6 items on the board and then remove 1 at a time placing 1 in each of 3 different groups. "We put 1 here, 1 there, and 1 over there." Then we remove 1 more and 1 more and 1 more until they are all used up. Since we are trying to regroup into sets of the same size we must always put the same number in each of the sub-groups. In 6 there are 2 in each of 3 groups.

$17 \div 5$ asks us to distribute 17 things equally among 5 people. We arrange 17 items on the hundred-board and then pick them off 1 at a time putting one in each of 5 different places. Now we do the same thing again putting one in each of 5 different places, then we do it again. Now we see that there are not enough to go around again so we have learned that there will be 3 in each of the 5 groups with 2 items remaining.

After some experience with the simpler problems students will see that it is possible to remove more than 1 at a time—they could put 5 in each sub-group if they had to divide 37 by 6 and they could put as many as 10 in each sub-set if they were to divide 59 into 5 equal parts.

Fractions

The relationships inherent in both common and decimal fractions can be exhibited on the hundred-board. By the use of elastic ribbons it is easy to show that $1/2 = 2/4$, that $1/2 = 50/100$, that $25/100 = 1/4$, that $1/5 = 20/100$, etc. Also that $1/10$ is equivalent to $10/100$ and that $3/10$ is equivalent to $30/100$. If the whole board represents unity or 100% then each square represents 1 out of the 100 or 1% and is useful in showing the common fraction and per cent equivalents. The board will be useful for revealing that $1/8$ of 100 blocks represents $12\frac{1}{2}$ little blocks. Since each block represents 1 out of 100 or 1% of the whole, then $1/8$ is equivalent to $12\frac{1}{2}\%$.

One column represents $1/10$ or .1 of the whole board and 6 rows or 6 columns represents $6/10$ of the board or .6 of it. Forty-three of the little blocks represent 43 of the 100 blocks or 43 hundredths of the whole. Thus were we to compare .4 with .43 we see that the .43 is larger than .4 by 3 little blocks or by .03 (3 hundredths). The board is a helpful adjunct in teaching the meaning of and the relative sizes of common and decimal fractions whose values lie between 0 and 1.

There are numerous other uses for the hundred-board which will be discovered by the teacher who uses one.

EDITOR'S NOTE. Dr. Volpel's "hundred-board" has many uses. Many people have used such a device for percentage but have not explored all the ideas he has presented. Perhaps he is right when he says it is the most valuable piece of equipment for enhancing the teaching of arithmetic. As with other devices, it is the way in which an item is used to foster discovery, understanding, and learning that makes it valuable. Teachers will see possible variations of design and construction and more values will be discovered as the use of the board becomes more common. Devices are valuable in leading children to sense the objective background for the learnings which later should become more abstract.

Postage Stamps and Arithmetic

A stamp collection, even a small one, may be a valuable source in connection with the study of arithmetic. Look for numerals that differ from ours—on the stamps of Lebanon, Pakistan, and Burma for example. An Italian stamp, commemorating the anniversary of Volta's invention of the galvanic battery, has the numerals MDCCIC and MCMIL. A few stamps have been issued in honor of mathematicians as the Greek series (G. 124M) in honor of Pythagoras. In 1959, Japan brought out a special stamp in honor of the formal adoption of the metric system in that country. It carries the numeral 10 with scales, a beaker, and a tape line to illustrate the measurement of weight, capacity and distance.

The Dual Progress Plan in the Elementary School

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At the recent meeting of the National Council of Teachers of Mathematics held in New York City,* one elementary school panel dealt with the problem of providing better instruction in mathematics to gifted students. Thus far, most attempts to meet the needs of gifted students have depended upon horizontal enrichment. A difficulty has been that most elementary teachers lack sufficient knowledge of mathematics to make enrichment programs successful. The mathematics preparation of elementary school teachers must be improved.

Equally important is the need for a new basic program in elementary-school mathematics that is appropriate to the needs of all students—the slow, the average *and* the gifted. Enrichment offerings and special classes for the gifted are at best piecemeal solutions.

The "dual progress plan" devised by George D. Stoddard, Dean of New York University's School of Education attracted interest at the panel referred to above. This plan is offered as a way of remedying the major shortcomings of current mathematics instruction in the elementary school.

Specialist mathematics teachers for elementary schools

It is a well-established fact that most common-branches teachers in the elementary school know very little about mathematics beyond the facts and tool skills of simple arithmetic. This forces them to rely unduly on texts and teachers' manuals. It prevents them from adapting their instruction effectively to the learning needs of individual students. Also, a good many common-branches teachers dislike mathematics and as a result are incapable of inspiring their students with enthusiasm for the subject.

* Christmas Meeting, December, 1959.

The dual progress plan, in common with other plans that departmentalize instruction, deals with this problem by having all mathematics in the elementary school taught by full-time specialist teachers. These specialists are selected from the ranks of elementary school teachers on the bases of their knowledge of mathematics and their interest in teaching it. No longer are there fourth-grade or fifth-grade teachers who teach arithmetic as just one of several subjects they must cover. Instead, the new plan calls for specialists whose knowledge of mathematics, and whose understanding of young children and how to teach them, qualifies them to teach all levels of mathematics that can be taught in the elementary school.

Nongraded advancement in elementary school mathematics

A second major shortcoming of the usual programs for teaching mathematics in the elementary school is the fact that students who differ greatly in their capacities to advance in the subject are held to the same rate of advancement through the curricular sequence. In the lock-step of the grade-level curriculum, both slow and gifted students in a given grade are offered the same segment of the curriculum as the year's work. The slow learner struggles and fails under the pressure to maintain grade level. The gifted student either learns patience or suffers boredom as he waits for the slower students to catch up with him, even though

he may be kept busy with "enrichment experiences" offered at his grade level.

The dual progress plan offers a solution by replacing the grade-level curriculum in mathematics with a nongraded program in which all students advance along the sequence of understandings and skills at rates that are suited to their individual learning capacities. In the new plan, there are no grade-level demands or grade-level restrictions. Slow learners are permitted to progress more slowly than the pace called for in the usual grade-level curriculum. They are given sufficient time to master a topic before moving on to the next. Gifted students, freed from the barriers set up by the grade-level course of study, may now learn in a single year the understandings and skills that slower students may require two or more years to master.

For nongraded instruction, students are assigned to classes whose members are at the same general level of advancement in the subject and have about equal abilities to advance further in the subject. Students are assigned to these classes without regard to grade placement. Crossing the usual age lines occurs whenever the student's level of advancement in mathematics places him behind, or ahead of, other students of his age.

The organization of instruction in the dual progress plan

A brief description of the total dual progress plan is needed to show the general framework within which mathematics teaching is conducted. In the plan, students advance in different curricular areas along two tracks, which is why it is called the "dual progress plan." The usual grade system applies to language arts, social studies, and physical education. Students are grouped into grade-level classes in language arts—social studies and receive two hours of instruction in this "core" area each day. The same classes have a period of physical education each day.

The nongraded system applies to mathematics, science, art and crafts, and music. Students receive forty minutes of instruction daily in mathematics and science, and forty minutes on alternate days in art and crafts, and in music.

In the new plan, *all* teachers specialize in teaching the subjects they know best and like best to teach. "Core" teachers conduct two two-hour classes in language arts—social studies each day. Physical education specialists teach a succession of 30-minute classes each day. Teachers of mathematics, science, art and crafts, and music teach five or six 40-minute classes each day. Except for replacements, all specialist teachers are drawn from the regular elementary school staff.

Currently, the dual progress plan is being tested in a three-year co-operative study involving the new Experimental Teaching Center at New York University and the school systems of Long Beach, Long Island, and Ossining, Westchester County, New York. The study is financed by a grant from the Ford Foundation. All students in grades three through six in the two school systems are included in the study. The Experimental Teaching Center is chiefly responsible for the research evaluation of the plan.

Preparing teachers to specialize in mathematics within the new plan

Some of today's common-branches teachers are better prepared to teach mathematics than others. It is safe to assume that, when an elementary school assigns all the teaching of mathematics to those members of its staff who know the subject best, and who wish to specialize in teaching it, the overall quality of mathematics teaching immediately improves. However, specialist teachers selected in this way need a good deal of further preparation for the job of teaching the full range of understandings and skills that may be taught in the elementary school. A major advantage of the dual progress plan is that it frees teachers to concentrate their in-service preparation in just one curricular area. And it encourages them to take further university courses in their chosen field.

In future, if specialist teaching of mathematics in the elementary school is adopted by school systems, schools of education may establish programs that prepare teachers specifically for this role. It is likely that such a teacher-education program would attract to elementary schools many able students, men as well as women, who now shy away because they do not wish to teach the variety of subjects required of the common-branches teacher. A program to train specialist teachers of mathematics for elementary schools should include a college major in mathematics as well as the needed course work in educational philosophy, educational psychology, child development, school organization and curriculum, teaching methods, and so on.

Conducting nongraded mathematics instruction in the elementary school

The dual progress plan calls for a single course of study in mathematics to guide the instruction of all students taught under the plan. While they are taught the same understandings and skills as gifted students, slower learners will advance less rapidly along the curricular sequence and will not advance as far during grade school as more gifted students. The new plan does not require the slower learner to reach any given level of advancement in mathematics during any given year, or before entering junior high school. Promotion does not depend on progress in mathematics, but only on progress in the language arts—social studies area. Freed of grade-level barriers under the new plan, some gifted students will progress to levels of advancement in mathematics usually taught in senior high school.

While the same curricular sequence is offered to both slow and rapid learners, and while both groups of students are required to master each learning task before proceeding to the next, it is obvious that there will be differences in how fully the two groups learn each task. Gifted students will obtain a more abstract understanding of the topic, and will be more facile in applying their knowledge of the topic to other areas of mathematics. Also they will be more capable of utilizing their knowledge of mathematics in new problem situations.

In teaching mathematics to any nongraded class, the teacher is challenged to individualize his instruction to meet the needs of different members of the class. The purpose of nongraded ability grouping is to make individualization of instruction easier to accomplish. Specialist teaching of mathematics should contribute greatly to individualization. When a teacher knows his subject well, and likes to teach it, he is prepared to give the expert attention to individual students that is needed if they are to realize their potentialities for learning.

Answers to critics of nongraded instruction by specialist teachers

Educators favoring the self-contained classroom have criticized the dual progress plan, claiming that it fosters "subject-centered" rather than "child-centered" teaching, slights the needs of the "whole child," and prevents the "integration of learning experiences" involving different areas of the curriculum. The first of these criticisms appears to be based on the assumption that a teacher who is expert in his subject will, because he is an expert, ignore the needs of his students as developing individuals. The opposite is likely to be the case. How can the teacher who neither understands nor likes mathematics meet his students' needs for learning the subject? It is the mathematics specialist who, knowing and liking his subject, is most free to concentrate his teaching on meeting the needs of individual children. It should not be forgotten that proponents of the self-contained classroom do not object to specialist teachers in art, music, physical education, and remedial reading. Why do their objections to specialization in the elementary school apply only to mathematics and science?

The notion that the specialist teacher ignores the "whole child" by teaching him just one subject implies that one can teach a

part of a child. Actually, when a student responds to any learning situation, he responds as a total person. There is no part of him set aside for mathematics, with other parts set aside for science, or English, or social studies. True, any departmentalized program presents the need for a child's teachers to consult with each other about the child's progress and problems. Special provisions are needed to insure that the child's total program is reviewed frequently.

The integration of learning experiences involving different areas of subject matter may be accomplished by the specialist teacher provided he has an interest in relating his curricular area to the solution of problems of living. Very often he can accomplish such integration better than the common-branches teacher since he knows his subject better and knows better how to apply knowledge of the subject in various situations. Many of these situations will call for relating mathematics to science or to social studies.

How well students learn in each curricular area with the new opportunities provided by the dual progress plan is being determined in the research study of the plan. The research test is not slighting the emotional and social needs of students as developing individuals. The new plan will be judged successful only if it is found to provide adequately for meeting these vital personality needs at the same time it provides for the intellectual advancement of students in accordance with their learning potentialities. Also, the research test will determine whether the plan's provisions for specialist teaching result in better qualified and more satisfied teachers.

EDITOR'S NOTE. Here indeed is an interesting departure from the usual school program. It places mathematics and science in the area of special study along with art and music. Pupils are promoted, in the usual sense, in terms of the "core" of social studies and language arts. They proceed at their own optimum rates through the subject matter of mathematics without regard for the normal grade-level standards. Here then is a combination of graded and ungraded school. Isn't this in essence what many schools are doing without openly admitting it? The procedures may differ and emphasis may be different from one school to another but many schools have found it desirable to establish rather elastic standards for certain of their pupils. The actual arrangement is less important than the ideal to have each pupil progress at his own best rate. Of course there are many school people who will insist that the "peer group" must be maintained. But what is a "peer group"? Peer in what respect? Who is to say that chronological age has more "peer status" than mental age or social sophistication? As one not-so-young observer of the educational scene remarked, "Let's quit listening to the highbinders and get on with the job of teaching children."

A Puzzle for Any Age

The following sequence will provide a person's age and his month and date of birth. Try it. Write down the number of the day on which you were born (20, for April 20); multiply this by 2; add 3 to the product; multiply this by 50 and then add the number of the month (4 for April) of birth. Next add 5, then multiply by 100 and finally add your age in years. From this number subtract 15500. The last two digits will be your age in years, the next two to the left will give the number of the month, and the next one or two to the left will be the day of the month in which you were born. Try it on a friend and then use it with some pupils. This will give some computational practice in an interesting setting.

Dozens and Dozens

Is there any difference between a half dozen dozen and six dozen dozen? Try it.

Squares

Show how you can cover half of a square two feet on each side and still have a square that is two feet from top to bottom and two feet across.

Arithmetic and "Block Work" in Primary Grades

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ARITHMETIC AND THE CONTENT FIELDS are frequently used in a mutually supporting relationship in the middle and upper grades. Arithmetic is used when it is needed in the social studies or science class, and vice versa. The content fields and arithmetic complement each other to the benefit of the children's understandings of both arithmetic and the content field. This is in accord with a major objective of arithmetic that children should be sensitive to number whenever and wherever it occurs and that they should develop the habit of using number effectively in such situations.*

In the early primary grades the block work periods offer comparable opportunities for integrated learnings. In general, there has been little recognition of the many naturally occurring situations in block work for the teaching of number concepts and arithmetical understandings. While social studies, language development, and social learnings are frequently cited values, the number learnings are less recognized.

The Nature of Block Work

What is block work and how is it taught? Typically the focus for the unit study is the community and the children's immediate environment. Most of the floor space of the classroom is used by groups of children who are busily constructing their chosen part of the community—a home, a store, garage, airport, hospital, etc. As the children gain experience and information about the community, the individual buildings and their relationships become more complex and more closely resemble the real community.

The blocks used in primary grades are of solid wood, in contrast to the hollow wooden blocks of the kindergarten and pre-school. The smooth solid blocks come in a variety of sizes and shapes which involve number vocabulary and some understanding of numbers. The teacher and children refer to the blocks by names, such as "long blocks," "half blocks," and "triangles."

Valuable number learnings may be taught while retaining the techniques which have already proved effective in teaching block work. While no basic changes in teaching are needed, there is a need—and that is for teachers who are alert to the inherent number opportunities. It will then be possible to use block work in achieving some of the purposes in the arithmetic program.

The purposes include:

1. NUMBER READINESS.

Mike had built the corral fence with a series of small long blocks. Realizing that he would need the same number of blocks when he built the fence again, Mike decided to remember the total number. He counted quietly to himself, "Two, four, six, ..."

The teacher asked, "How many, Mike?" When he answered with the total, she continued, "How many two's is that?"

"Hunh?" said Mike. "How many two's —Oh, sure!" and as he proceeded to count his groups of two fence rails, Mike had moved a step toward readiness for a new process.

* National Society for the Study of Education, *The Teaching of Arithmetic*, The Fiftieth Yearbook, Part II. Chicago, Illinois: The University of Chicago Press, 1951. P. 7.

2. NEW USES OR APPLICATIONS FOR PRESENT NUMBER UNDERSTANDINGS.

The first grade had discussed *one* and *one-half* and had manipulated bright felt circles on the flannelboard. A short time later Janis was making a wall. She did not have enough long blocks, so she was fitting half-blocks in the place of the long blocks which she preferred. As she worked she was mumbling half audibly, "Half . . . an' half . . . an' a long . . . Boy! this's the same. Boy! half-half an' *one*."

3. NEW CONCEPTS.

Phillip, bracing the roof of his gas station with cylindrical blocks, said, "These big, fat posts sure hold it up."

The teacher, who was watching him and listening, said, "Yes, the posts are strong. If you were using them for a storage tank for gasoline, would you call them *posts?*"

Phillip sat back on his heels and slowly said, "No,—but I don't know what."

"What is *different* when you use this block," pointing to a post, "as a storage tank?"

"Oh, yeah! What it holds."

4. NUMBER VOCABULARY.

The preceding illustrations have shown how number vocabulary is developed and used in the process of children's work with block building. Vocabulary that is developed meaningfully and used accurately serves in the immediate situation and as a foundation for later arithmetic experiences.

The variety of sizes and shapes of the blocks leads to growth of number vocabulary.

Number Situations with Blocks

What kinds of number situations arise in block work? What are some number understandings and skills that can be taught? The following examples from classrooms will indicate some specific answers to these questions and, it is hoped, may stimulate teachers to other applications in their own teaching and with their own classes.

COUNTING

One of the first problems that arises in block work is, "How many blocks should each person carry?" Many children are simultaneously taking the blocks from their storage place on shelves or in cupboards and are carrying them to the chosen work space on the floor. Anxious to have enough blocks for their building, the children try to carry large armfuls of the slippery and unwieldy blocks. The teacher stops the activity and discusses with the children how many blocks to carry. "One" and "two" take on added significance with relation to size when the decision is made that "one long-block or two of other kinds is a load."

Jean had carefully lined up the blocks for one wall of the structure she was making and she was ready to build the opposite wall. The question she faced was, "How many will it take for the other side?" She counted the first wall and built a matching one. Similar counting and matching situations occur frequently for both height and length.

HEIGHT

As the buildings take form, problems of height occur. The teacher and the class discuss, "How high should the walls be for our house?" They decide that five blocks high is high enough to be satisfying but not unstable nor apt to fall easily. Before long the children so easily estimate "five blocks high" that few comments about it are heard.

At another work period it was agreed that some buildings, because of their functions, needed to be higher. A lighthouse at the harbor or a control tower at the airport were buildings that must exceed the building code of "five blocks high."

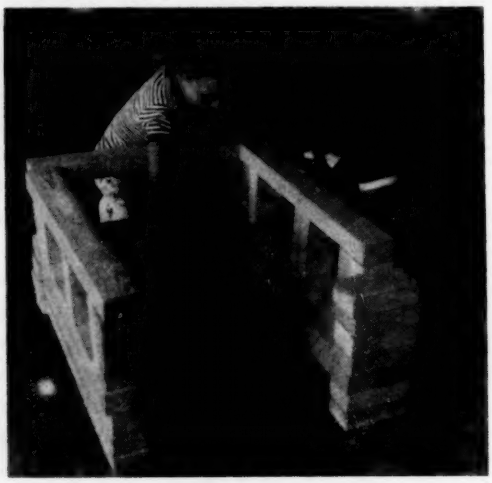
Another problem involving height is that of the placement of windows in the building. At first it is common for the children to build the walls with four solid rows of blocks and to leave spaces for windows in the top row of blocks.

"Can people see out of the windows in their house?" asks the teacher.

"No! No, the people can't see out."

The teacher continues, "How high should the windows be? How can we decide how high the windows should be?"

The children measure with the figures representing people and then decide that the windows should begin in the second row.



"How high should the windows be? Can our people see out?"

WIDTH

Width is another measurement concept which is developed in block work. "Is the door in your garage wide enough for the truck to go through?" the teacher wonders. Victor and Juan agree that it is not wide enough and that they must make some changes. They decide that the use of fourth-blocks in place of half-blocks will widen the doorway.

AREA

Other estimates of size are made in situations involving area. Attention is directed to the relative areas used in building parts of the airport by questions, "Is the landing field large enough for all of our airplanes?" "Should the hangar and the landing field be the same size? Which should be larger? Which should be smaller?"

When one group of children was building a house, the teacher inquired, "Are the rooms in the house large enough for the furniture? Is there room for the furniture and for the people to move around?"

The area which each group uses in the classroom is another problem. One day a committee was building the railway station and extended the railroad tracks into the areas where other groups were building. A few sharp protests showed that the class was dissatisfied with this behavior. The resulting discussion worked out solutions for the use of floor area. The decisions reached were that each group should build within its own floor space and not infringe upon that of others without their consent. It was further agreed that not all buildings required the same area—some needed more while others needed less.

FRACTIONS

Long blocks are preferred by the children wherever they can be used. However, when the supply of long blocks in the cupboard is exhausted, they must substitute with enough shorter blocks to equal the desired length. The children quickly learn that two half-blocks will equal one long-block, but they are slower to substitute the fourth-blocks. Because a roof is best made with long blocks, the children take the long blocks from the walls and replace them with half-blocks or fourth-blocks. If there are still not enough long blocks to complete the roof, the children trade for the needed blocks. In the exchange the children decide

the correct number of half-blocks and fourth-blocks that are equal to the desired number of long blocks.

Ordinal and Cardinal Numbers

Both ordinal and cardinal numbers are used as the block work progresses and the children gain understanding of them with ease. An example of ordinal counting, in which number names are used to arrange objects in order or to identify their place in a series, is, "What kind of blocks shall we use in the *first* row? In the *second* row?" The use of number names in serial order to find the total number, which is cardinal counting, is used in such situations as, "How many posts have we used in this side of the fence? Let's count: one, two, three, etc."

Block work in the primary grades provides children with opportunities to use in new situations the number understandings which they already possess, to develop new understandings, and to increase a functional number vocabulary. The integration of number learnings with unit work strengthens children's arithmetical understandings, makes them sensitive to number in social situations, and develops the habit of using numbers in such situations. Primary teachers find many opportunities in the course of the usual block work to integrate number with other content fields.

EDITOR'S NOTE. In most types of "activity work" with pupils the arithmetical values depend in large measure upon the discernment of the teacher and the way in which she asks questions and leads the thinking of the individuals in the group. In the modern school this activity serves a real educational purpose which is very different from the pretty birds and flowers that primary-grade children stitched some thirty years ago. Concepts of size and shape and of "fitting together" as well as many ideas of number can be found in many constructions with blocks such as are used in Los Angeles. Let us keep our eyes on the goals for arithmetic as well as on the social factors and the learning to adjust. As the authors say, "to integrate number with other content fields."

Council Resources for Arithmetic Teachers

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THROUGH THE COLUMNS of this Journal I wish to speak to those who teach arithmetic. I welcome this opportunity for it is my considered judgment that the quality of instruction in arithmetic is in large measure responsible for choices made in later years when mathematics becomes an elective. It is in the elementary school that students are first introduced in an informal manner to mathematical concepts and one of the purposes of the National Council of Teachers of Mathematics is to serve its members in such a manner as to improve the quality of this initial introduction. One measure of the degree to which the Council is achieving this important purpose is the extent to which its facilities are used by those who direct the activities of the arithmetic classroom.

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When a child walks into your class at whatever level of development, he brings with him some understanding of a number of mathematical concepts. Your responsibility as a teacher is to nourish the healthy growth of these concepts and to introduce him to others which may be needed in the building of mathematical structure. The natural numbers, for example, seem to satisfy his needs where counting only is involved but the concept of number must be extended to handle situations which call for measurement. The need for fractions is thus recognized, a need which is further emphasized as the student is guided to discover the significant fact that the operation of division is not always possible in a number system which includes the natural numbers only. The extension of the number concept to include fractions removes this limitation and the further extension of the concept to include the signed number makes subtraction always possible.

The preceding illustrations reflect what is meant by the growth of a mathematical idea and to provide for this kind of continuity is to emphasize the structure of mathematics, to involve students in the building of this structure and to encourage creative learning. It is this emphasis which is constantly reflected in the splendid articles of THE ARITHMETIC TEACHER. It is this thread of continuity which unifies the ideas in the three yearbooks of the Council, Nos. 10, 16, and 25,[1] which are devoted entirely to the teaching of arithmetic. The mathematics curriculum for kindergarten through grade 12 is, in fact, developed around selected continuing themes, all of which are defined in the 24th Yearbook under the suggestive title "The Growth of Mathematical Ideas." Are you familiar with these ideas? Are you teaching arithmetic so as to promote their steady and continuous growth? Helpful teaching procedures for all levels of instruction are suggested in the 24th Yearbook and there is perhaps no other publication of the Council which more clearly defines the very significant role of the elementary teacher in the growth of understandings associated with mathematical structure. The services of the Council are planned to help you meet this important responsibility.

In addition to the Yearbooks and the three Journals, which include THE MATHEMATICS TEACHER and THE MATHEMATICS STUDENT JOURNAL, some of our smaller publications would be especially helpful. What, for example, is 2? Is it a name given to the cardinal number of a group or is it a position on the number scale? Perhaps it is the value of "a" in the rational number "a/b" or it might be the value of the repeating decimal 1.9999999999 . . . and on to infinity. On the other hand it could be a short method of writing the complex number $2+0i$. If you, then, are asked the apparently simple question "What is 2?" the nature of your response will depend on the extent of your vision down the long continuum of the growing concept of number. The answer of Lawrence A. Ringenberg in his delightful little pamphlet entitled "A Portrait of 2" is the answer of a man who fully appreciates the rich significance of the oft quoted statement that "The perfected number system is, in many respects, the greatest achievement of the human race." This attractive pamphlet is but one of our smaller publications which will be helpful and stimulating to you. Others may be equally helpful as you guide your students in the growth of ideas leading to mathematical maturity.

[1] Yearbook No. 25 will be available by April, 1960.

Challenging the Rapid Learner

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ELEMENTARY SCHOOL TEACHERS have been charged with the problem of further developing the gifted or talented child in arithmetic. This is an area in which many have done something but in which no one has done enough. The following suggestions are not offered as a panacea for the situation but as practical ideas that may be used and from which others may be developed. In this way it is hoped an additional avenue will be opened for creative thinking in this area. It is intended that these suggestions will be beneficial to any teacher who in some way offers differentiated assignments for the more adept in arithmetic. These procedures should be of assistance in programs utilizing homogeneous sectioning, special teachers, sub-grouping, acceleration, or enrichment.

At the pre-school level the talented child is challenged by showing off before adults, by his adeptness at counting, or dealing with simple processes. At the high school level students are frequently allowed to take additional or advanced courses. Sometimes they are allowed to start on the so-called college level material. It is not the purpose of this article to argue the rightness or wrongness of the above practices but to show that tangible effort is being presented to the public and to emphasize that we do not do much that is comparable or recognizable at the elementary school level. It is the intent, however, to offer possibilities for challenging the talents of children in the elementary school.

If the inductive method of presentation is used in general teaching, elementary teachers usually do a commendable job of challenging the talented youngsters in getting ideas and answers pertaining to the topic being initiated. After this phase all the youngsters are too frequently sent over the same obstacle course. Teachers often require that the child with insight and mental adeptness place on paper the same algorisms as the slower learner rather than allowing him to skip steps after he has demonstrated understanding. In the "good old days" the reckoning of the talented was recognized, today teachers may be wrecking rather than recognizing this ability.

The suggestions which follow allow the use of the same material that is offered by our basic texts and which is usually used to teach for what has been termed mediocrity. We certainly do not wish to rule out a basic series as this is extremely important in a planned sequence of learning.

In the primary grades when children are expected to count by 3's, for example, we are generally satisfied with the traditional 3-6-9-12 approach. Why not have the rapid learner also count either 2-5-8 or 1-4-7?

Shorter Methods of Computation

In column addition of two place numbers as

$$\begin{array}{r} 21 \\ 36 \\ 27 \\ 43 \\ \hline \end{array}$$

we can have children add 57, 84, 127 or zigzag down 27, 57, 64, 84, 87, 127. All children, slow, average or rapid will work with the same material and arrive at the same answer but use different modes of approach. One may wish to start with an example in which the total of the right hand column totals less than ten or totals ten or more only when the last numerals are added.

Two three-digit numbers can be added in one operation and then expanded into adding a column of three digit numbers. After becoming proficient in the addition of two three digit numbers children quite naturally fall into doing the inverse, namely subtracting one three digit number from another.

When multiplying a two digit number by a two digit number one may arrive at the answer by jotting down the units resulting from the product of the ones; any "carried" ten is added to the cross product of the ones and tens and placing the resultant tens in the tens' place in the product; any "carried" hundreds are added to the product of the tens and then writing the result in the hundreds' place or to the places to the left if the product is greater than nine. Example:

$$\begin{array}{r} 23 \\ 42 \\ \hline 966 \end{array}$$

the product of 3 and 2 is 6, write 6; the sum of 2×2 (4) plus 4×3 (12) is 16 write the 6 in the tens' column; 2×4 plus the "carried" one is 9, write the 9 in the hundreds' column. A more difficult example is

$$\begin{array}{r} 87 \\ 65 \\ \hline 5655 \end{array}$$

think 35 write 5 remember the 3; think 40 plus 42 plus 3 is 85 write 5 remember the 8; think 48 plus 8 write 56. The child should prove his answers in the normal way either by multiplication or by division. He should also be encouraged to figure out why and how this process works. There are special situations which will work more rapidly when multiplying by numbers ending with 5 or when multiplying by a teen number should special students wish to delve further. The above will work in all situations, however. If students have difficulty with the total mental process they may write down the partial products as they arrive at them. This will still allow a challenging variation. It should also point up the fact that algorisms are merely crutches and we write only those steps which we cannot mentally retain.

In doing processes like the above and the following, the teacher must not be concerned with the situation where the children may become more adept and rapid at arriving at solutions than he. To build up our ego we can as teachers write off this adroitness as the effect of practice rather than admit the child's greater facility for a particular procedure or even mathematics in general.

When multiplying any number greater than two digits by eleven, imagine zeros at both ends of the multiplicand. Begin at the right and add each pair of digits in turn, together with any digit carried and write the results in the product.

This Procedure May Be Extended to Other "Teen" Numbers

Example:

$$\begin{array}{r} 2684 \\ 14 \\ \hline 37576 \end{array} \qquad \begin{array}{r} 026840 \\ 14 \\ \hline 37576 \end{array}$$

Think 4×4 (16) plus 0 to the right is 16, write the 6 "carry" the 1; 4×8 (32) plus 4 to the right (36) plus "carried" 1 is 37, write the 7, remember the 3; 4×6 (24)+8+3 is 35, write 5, remember 3; 4×2+6+3 is 17, write 7; 4×0+2+1 is 3, write 3.

In so-called "long" division the pupils may write only the remainders in the partial product as an intermediate step toward the "short" division. Example:

$$\begin{array}{r} 985 \\ 8\overline{)7880} \\ \leftarrow 68 \\ 40 \end{array} \qquad \begin{array}{r} 656 \text{ r } 10 \\ 53\overline{)34778} \\ \leftarrow 297 \\ 328 \end{array}$$

When dealing with the division of a fraction by a fraction it is generally conceded that the gifted children are capable of arriving at the reason for inversion of the divisor and teachers should work toward this end. There are several approaches to the situation and the teacher should have an acquaintance with all of them and rely on the group for the one that seems to yield the best re-

sults. After the children understand the concept of a proper fraction as a divisor and the reason for inversion, the following procedure is suggested: In solving an algorism such as $5/6 \div 2/3$ the child should eliminate the inversion step and simply cross multiply using the dividend as the clue for numerator and divisor and arriving at $15/12$ or $1\frac{1}{4}$. True this could be given to the average learner or even the slow learner through the rule and drill approach but it would be deemed very unadvisable.

Another manner of dealing with specialized situations involving fractions is frequently referred to as the "double and halve" method with variations. It is usually used when multiplying a whole number by a fraction but can also be used when dealing with two whole numbers or two fractions. Examples:

$$82\times 3\tfrac{1}{2} = 41\times 7$$
$$440\times 2\tfrac{1}{2} = 220\times 5 \text{ or } 110\times 10$$
$$660\times 3\tfrac{3}{8} = 220\times 9$$
$$84\times 1\tfrac{1}{4} = 21\times 5.$$

Here the children can do research or inductively develop rules governing when and if a number is divisible by certain numbers such as 2, 3, and 5.

One Procedure Checks Another

Some of the above processes either are a check for the regular work or the normal processes are a check on the special procedures mentioned. Another method of checking which can be used by the rapid learner is the "excess of nines" method (frequently referred to as the casting out 9's method). Any teacher who is not acquainted with this method should consult his high school mathematics teacher or some text on arithmetic or mathematics backgrounds for teachers. This check is fairly accurate but not infallible as it will not pick up transposition errors or some compensatory combinations of nine. For the exceptionally talented the check by "excess of elevens" may be used. These checks function in all four basic processes.

Another check that may be used for multiplication and division is the use of the slide rule. This is one device that not only outwardly challenges those highly interested and competent in arithmetic but also sets the stage for demanding a much greater understanding of place value in our number system. Slide rules which are adequate for this use may be purchased for as little as 50¢ each. Frequently rules may be borrowed in the community. If the child owns the rule he may proceed into more advanced work such as the squaring of numbers or of finding square root.

The area of the historical aspects of the processes being studied should not be left out. The talented student should be encouraged to find out how each process was evolved and be able to work with several of the older methods of computation. The lattice method of multiplication or the galley method of division are two that should provoke much interest, improve understanding, and challenge thinking. Parents may be used as resource people to show different ways of subtraction. The work that the talented do in research in these older methods can well enhance the arithmetic program of the average and slower learner if the results of research and study are ably presented to the total class. The major objective may be the improvement of interest for the total group rather than for understanding. This will also tend to tie the group more closely together and all will benefit from the efforts of several. The writing of story problems, to stimulate the thinking of either the rapid learner or of the total class, is an area that can utilize capabilities usually not touched. It also provides material which will supplement the basic series.

In all of the processes mentioned the talented child should be encouraged to do as much non pencil-and-paper work as possible and to take as many shortcuts as he can understandingly explain. The talented child should, in estimating answers, be expected to work with less tolerance or deviation than would be expected in the normal program.

(*Concluded on p. 316*)

The Abacus—A New Use for an Old Tool

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ARE YOU LOOKING for an interesting approach to the understanding of number concepts? Does your class completely understand the use of the zero, place value, and what we mean when we say that our number system has a base of ten? How well do you, as a teacher, understand these concepts? How well can you put forth these ideas to your class?

In our sixth grade class in Glencoe, Illinois, several types of Oriental abaci were introduced. A short history of the abacus was presented to the children, followed by a demonstration and a discussion. We hoped at the time this was presented to stimulate from 6 to 10 children to the point where they would be interested in learning how to use the abacus themselves. At first the children worked with colored squares at their desks. We found a nucleus of children with keen interest in the abacus who considered it a fascinating device with which to compute. What followed was an abacus club, which met during recess or any other free time the children could find. Many of the children made their own abaci. This can easily be accomplished with cigar boxes, string or wire and large beads.

There were some interesting arithmetic activities taking place in class at this time. We found the children were having fun working arithmetic problems by themselves. They enjoyed racing with each other to see if they could work a problem faster on their abacus than their neighbor could on a piece of paper. They would get together at each others' houses after school and on week ends and to the amazement of their parents would work arithmetic problems for hours. We then proceeded on to multiplication problems and several children even taught themselves how to divide on the abacus.

It should be made clear that our purpose was not to develop proficiency with the abacus. To a slight extent this came as a result of the children's own interest. We were much more concerned with the fascination the children showed for numbers and with the increase in arithmetic skills which resulted. Their repeated working with numbers had a beneficial effect upon their speed of computation of the arithmetic combinations on paper. They also gained new insight into the borrowing and carrying concepts of arithmetic. This in itself was sufficient to make the time spent with the abacus worthwhile. The greatest results were to develop later.

It is difficult to appreciate the feelings these children had towards numbers and the above mentioned understandings without having the experience of using the abacus yourself and teaching its use to children.

The natural interest of the children led to a further study of our number system. We took another look into our number system with its base of ten. We started with the number chart much the same as the children did in the primary grades (Figure 1). After a careful study of the number chart

FIG. 1

OUR NUMBER CHART—BASE 10

100,000	10,000	1,000	100	10	Units
			1	1	1
			2	2	2
			3	3	3
			4	4	4
			5	5	5
			6	6	6
			7	7	7
			8	8	8
			9	9	9

the class made two significant generalizations, the first being that each column is a multiple of the preceding column. In our number system it is a multiple of ten. The third column instead of being shown as 100 could also be shown as 10×10. The fourth column, instead of 1000, could be $10 \times 10 \times 10$. At this time we showed them how mathematicians preferred to write a number repeatedly multiplied by itself, i.e., 10^2 and 10^3. The second generalization was an awareness that each column could only have nine symbols and that the presence of a tenth symbol would automatically place some value in the next column to the left. These were actually not new concepts to the children but through the use of the abacus they took on new meaning. We could then generalize the logic of our number system and replace the column heads of units, 10^1 10^2, 10^3 with any symbol. Let us use X as the new symbol. The column heads would then be units, X^1, X^2, X^3, etc. This then is the formula that illustrates the logic of our number system. The children were capable of following this explanation. We then applied the formula to another number base other than ten. We used the number base of two in class. To show how this would work let us apply the formula. In place of the X's substitute 2. The column heads will have a value of units, 2^1, 2^2, 2^3, or 2, 4, 8 (Figure 2). In each column is allowed only one digit. The class then learned how they might count, add, subtract, multiply and divide with this number base just as they had done using the base ten (Figure 3). Counting and computation in a system based upon 2 or any other number gives children a new experience in working with the zero and the concept of place value. Since in the binary system only one digit is allowed, the place holder, zero, is used frequently.

FIG. 2

BINARY NUMBER CHART—BASE 2

2^5 or 32	2^4 or 16	2^3 or 8	2^2 or 4	2^1 or 2	Units
		1	0	1	1

On this binary number chart the number eleven is represented.

FIG. 3

ADDITION AND SUBTRACTION—BASE 10 AND BASE 2

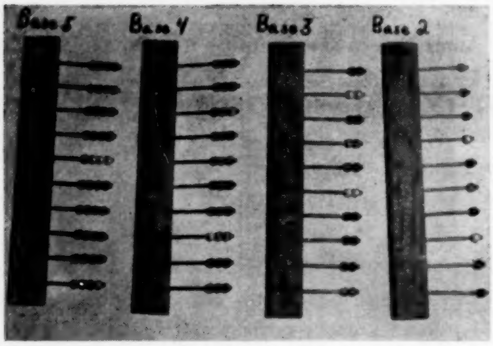
Base 10	Base 2
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000
9	1001
10	1010

	Base 10	Base 2
ADDITION	7	111
	+2	+10
	9	1001
SUBTRACTION	9	1001
	−5	−101
	4	100

The same logic can be applied to any base used. Another example we used in class was the base 5. The column heads being units, 5^1, 5^2, 5^3, of units, 5, 25, 125, etc. In each column we have four digits. At first the class used the regular digits of 1, 2, 3, 4. Later they realized they could make their own symbols such as # for 1, & for 2, % for 3, ! for 4 and so forth. We then apply the logic of this formula of our number system to any number we may wish to use as a base.

Abaci can be constructed with number bases other than base ten. Children are able to count, add, and subtract on these abaci.

Although the abacus was introduced in the classroom as a means of motivating children to improve their arithmetic skills, it is obvious that the benefits were far greater. The zero as a place holder took on new meaning. The class could see our number system as a system based upon logic and order. They were able to appreciate mathematics as a science. Although they worked hard to understand the concepts presented



Homemade abaci with various number bases.

they enjoyed the experience and gained a feeling of accomplishment.

We are convinced that the abacus can be used successfully as a teaching tool in arithmetic. It is our feeling that studies should be made to seek answers to several questions. At what grade levels should an abacus be used? How deeply should each grade level delve into our number system? Can the teaching of generalizations and formulas play a more important role in the teaching of arithmetic in our elementary and junior high schools?

If you as a teacher, or your school district should engage in experimental work with the abacus you will find the results most rewarding.

EDITOR'S NOTE. The abacus is certainly an old device which has been used many centuries for computation. However its historical usefulness was associated with number systems that differed significantly from the Hindu-Arabic system which we use. As the authors point out, much can be learned about our number system by using an abacus. The open-end sticks on a frame seem most useful because this one frame can then be used for work with any number base. The basic principles are the same with various bases and it is the learning of these principles that is important. Later, in work with pencil and paper the principles of the number system are useful in understanding what is being done with numbers and is discovering short cuts and easier procedures. The abacus can be and should be much more than an "interest device."

The Rapid Learner

(Continued from page 313)

As an example, in a problem situation which might utilize the multiplication of $12.98 by 75 we might expect an estimate of $975 rather than an estimate of a little over $900. The climate of expectation should be changed for the more capable in the mathematical area.

The resources of our library and the activities of our everyday living can invite many challenges. The most important asset in developing material for the rapid learner is to have the teacher believe that something valuable can be accomplished. The next important aspect is to direct the thinking and energies of children in the direction of doing extra and interesting challenging supplementary activities. To these two add a little time, enthusiasm and commendation for critical thinking and many new ideas can be developed and a self-challenging program will result.

EDITOR'S NOTE. Dr. Schwartz has given a number of special procedures which may be used to challenge the more able pupils. His examples are largely in the computational area but he asks for both understanding and appreciation on the part of the pupil. It would be a mistake to teach many of the short cuts to any of our pupils to be learned and used mechanically without an understanding of what is being done. On the other hand, certain special procedures are very rich in their potential for developing understanding and insight.

We may use many types of enrichment material for our able pupils. These young people should be able to think well and this is an aspect of learning upon which we should capitalize because thinking will continue as one of our main objectives. As the author points out, "the most important asset in developing material for the rapid learner is to have the teacher believe that something valuable can be accomplished." Let us all make progress in this direction.

Prognosis for Studying Algebra

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Who should study algebra and when should this study begin? With the renewed interest in the study of mathematics and since the program of the ninth grade in many schools continues to be mostly algebra it is pertinent to consider in grades seven and eight the prognosis for success in algebra. This problem was investigated by the writer and the conclusions he found are briefly presented in this article.

The writer made a detailed two-year study of his algebra classes at Culver City (148 cases each year), and derived a prognosis battery with a multiple coefficient of correlation of .78 for the first year, and .86 for the second year. The standard error of these multiple correlations is around .03. With these standard errors we are confident at the 1% level that the "true" correlation would be within $\pm$.08 of the obtained correlation. The first-year results would then put the "true" multiple correlation between .70 and .86, and the second-year results between .78 and .94.

This overlap of ranges for the "true" multiple correlation would further restrict the range of the "true" correlation to the range .78 to .86. Based on the combined studies, we can then be confident at the 2% level that further computations of multiple correlations based on this same population with the same prognosis and criteria variables would give results within this same range—.78 to .86—a range high enough to be very helpful in prediction.

Student success was based on scores on standardized algebra achievement tests. Useful prognosis variables included algebra prognosis tests, I.Q., arithmetic achievement tests, and previous arithmetic class grades.

Selection of Prognosis Variables

The writer, having taught algebra classes for about ten years, had various guesses as to what measures would predict which students were most likely to succeed or most likely to fail in algebra. To measure success at the end of the semester he used the average of the scaled scores on two forms of the *Seattle Algebra Test*, and at the end of the year the average of scaled scores on two forms of the *Cooperative Elementary Algebra Test*. To predict success he naturally thought of I.Q. (*California Test of Mental Maturity*), *Orleans Algebra Prognosis Test*, previous teachers' grades, and arithmetic competency (*Cooperative Mathematics Test for Grades 7, 8, and 9*).

He was disappointed in other variables he thought might be pertinent such as "Number" and "Reason" on Thurstones' *Primary Mental Ability Test*, chronological age, sex, and an attempted measure of "willingness" and "pessimism" based on the student's own opinion at the first of the year. Along with these he discarded the parent's written opinion as of the first of the year, a composite of recorded work-habit grades from the previous year, and reading grade placement.

Since the *Orleans Test* is a test that is tedious to time and slow to score, the writer constructed a 24 item multiple-choice prognosis test to see if something easier to use could be devised. He also attempted to write it so that brief exposure to algebra would cause less change in the scores. Surprisingly, this twenty minute *Pretest* correlated almost as well with semester success as did the *Orleans* (.57 against .64 for the first year, and .69 against .72 the second year). Comparison was closer for the end-of-year success (.64 against .65, and .69 against .76).

To distinguish the useful variables from the maze of thirteen variables, the Wherry-Doolittle Test Selection Method was employed. The first variable selected by this method was the 80-minute *Cooperative Mathe-*

matics Test, then was added the I.Q. as measured on the *California Test of Mental Maturity*, then came previous year's arithmetic grades, and finally the scores from the Orleans test.

Second-year Findings

The following year the author again had all five of the algebra classes at Culver City Junior High School, with again complete statistics on 148 students. For economy of time only the five scores (*Cooperative Mathematics*, I.Q., arithmetic report card grades, *Orleans*, and author's *Pretest*) were retained.

The *Pretest* was doubled to 48 items by writing a second 24 items closely following the outline of the first 24. The original 24 item *Pretest* had a matched-half coefficient of reliability of .75. Statistical theory predicted that doubling the test would give a reliability of .85. Actual obtained results showed the 48 item *Pretest* as having just over .83, which was very close considering that different student populations were used.

The Doolittle method was used to compute the multiple correlation coefficient. This gave a multiple correlation of .86 between the five criteria variables and the average of two forms of the *Cooperative Algebra Test*.

"Pretest" Used for Screening

These results encouraged the guidance department at Culver City Junior High. They decided to give the *Pretest* to all eighth grade students soon after the middle of the year, in time to use the results in screening for algebra. Rather than use complex regression coefficients to produce a prognosis battery score, they used an informal composite of *Pretest*, I.Q., and teacher recommendations. The general thinking was that if a student was low in as many as one of these criteria, there would need to be some compensating strengths indicated elsewhere in his total guidance record to justify letting him try algebra at that time.

The third year, 173 students took algebra. A correlation of the scores on the 48-item *Pretest*, administered in March of 1956, with report card grades in algebra in June of 1957 showed a coefficient of correlation of .66. In the examination of the results, it was found that no student with a score below 31 (based on 48) received an A, no one with a score below 25 received A or B, and no one with a score below 19 received a C or better. Since the teachers were not aware of the pretest scores, they were not influenced thereby in awarding algebra marks. Culver City has been using the *Pretest* ever since with good results. The writer did not teach any of the classes in the study after the second year since he was transferred to the senior high school.

Application of Results to Other Schools

The use of the test in Culver City suggested the possibility of making the *Pretest* available to other schools. As a result, the *Pretest* was extended to 72 items from which the best 60 were selected by item analysis techniques based on scores of students from several schools. This improved form showed a coefficient of reliability of .91 for an eighth grade group and .93 for a ninth grade group. The final form of the 60-item test has been published under the title of *Survey Test of Algebraic Aptitude*. Since the test is a 40-minute multiple-choice test without correction factors, its ease of administering and scoring is evident. It can be used individually or in large groups.

The writer wishes to make a closing caution on test usage. No single test can be used blindly. A test with correlation of .71 accounts for only 50% of the variable quantities needed to predict absolutely. A correlation of .90 would account for only 81% of the variables. This indicates that we cannot absolutely disqualify any student from "trying" algebra in a public school. But, with such measures as the *Survey Test of Algebraic Aptitude*, I.Q., previous grades in arithmetic, and teacher ratings we are in a position to warn parents and counsel students when extreme effort and/or outside

help are mandatory if the student is to survive academically in algebra.

In Culver City, the screening has been done a year earlier on some superior students in selecting those who now start their regular college preparatory algebra in grade 8 instead of grade 9. This is consistent with the writer's study for the master's degree since chronological age had such a small correlation with success (about $-.25$), indicating that age is not a serious factor.

In explaining the scores to parents, who seem to respect test scores more than teacher opinions, scatter-diagrams are very helpful. If these plot the actual ranges of final grades that have resulted for each prognosis score, it seems to have more meaning to parents than does a maze of technical terms familiar only to the students of statistics.

EDITOR'S NOTE. The role of counselling and advisement of students is very important. Certainly advisement should be based upon the best information and judgment available. Mr. Dinkel has sought for the factors which are most pertinent and has produced a test of prognosis in algebra. His correlations of scores on this test with later marks in algebra are fairly high in such a testing situation. He wisely points out that other factors such as previous marks in arithmetic, the judgment of teachers, and the will to work are also important in making the final decision. Since a student's future may be greatly influenced by counselling it is imperative that this be done by people who have a good deal of insight and understanding. It is doubtful if educational guidance should ever be based upon the results of one test.

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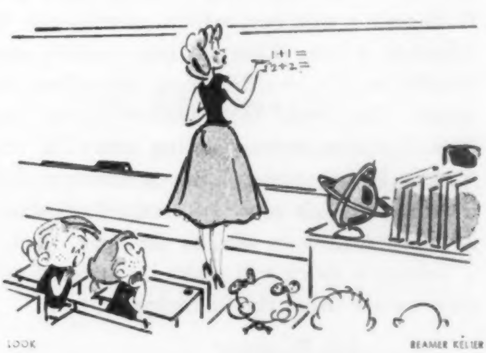
Cup Cakes in Kindergarten

Kindergarten and first grade children may record attendance with the paper cups in which individual cakes are baked. Use blue cups for the boys and pink ones for the girls. Call the boys and girls your Cup Cakes.

Suppose there are fourteen boys in a class. Ten will stand in a row, and four will stand in a row apart from the others. The number of boys is ten and four, and we call this fourteen. A girl in the class pins a blue cup on the bulletin board for each boy, with ten cups in one row and four cups in a row apart. The number of cups is ten and four, or fourteen.

Attendance for the girls is taken and recorded similarly.

When pupils are a little older, the whole class can be lined up in tens and ones and the attendance recorded with paper cups.



"I must be growing up. She's beginning to look good to me."

Courtesy of Look. *Not to be reprinted without further permission*

The Madison Project*

Marie Lutz
Syracuse, New York

"Mrs. Downing, why didn't you teach me signed numbers the way you are teaching them to my brother? He knows them better than I do! He helps me!"

In this way, a ninth grader recently testified to the success of the Madison Project now being inaugurated in one of the seventh grades in his school.

Perhaps the most important contribution of the Madison Project is in the field of motivation. It captures the children's "will-to-do" and before they realize what is happening, they are using signed numbers as though they had been acquainted with them for a long time.

Consider the matrix game. The score can be 0 (tie), minus (one side) or plus (the other side). The score is computed after each play, thus insuring a quick, easy use of the addition of signed numbers.

In the true manner of the scientist, why don't you try the matrix game so you can judge the results for yourself? In this game there are two sides: side A, and side B. To play the matrix game, side A will choose a letter, which corresponds to selecting a column in the following matrix, while side B chooses a number, which corresponds to selecting a row. They do this secretly and simultaneously, writing their selections on paper. They read their choices aloud and then find the corresponding entry in the matrix. If this entry is a minus number, side B wins. If it is a plus number, side A wins. Zero (0) means no win, or tie score.

Trying it twice will make it clearer. Suppose we are using the following matrix:

Side B's choices / Side B wins if score is minus (−)

$$\begin{array}{c} \\ \text{“1”} \\ \text{“2”} \\ \text{“3”} \end{array} \begin{array}{c} \begin{array}{ccc} \text{“A”} & \text{“B”} & \text{“C”} \end{array} \\ \begin{bmatrix} +1 & -1 & 0 \\ -2 & 0 & +2 \\ +1 & +1 & -2 \end{bmatrix} \end{array}$$

Side A's choices / Side A wins if score is plus (+)

Suppose, for his first move, side A chooses "C", while side B chooses "3". Then the last row and last column have been selected, so the corresponding entry is -2. In other words, B has won this round, and is now ahead by two points. We write this score as -2.

Now it is the second turn. Suppose side A chooses "C", while side B chooses "2". When they read their choices aloud they will realize that they have chosen the third column and second row. The entry in the third column and second row is $+2$ (you may prefer to read this "positive two," and write it $+2$). This means that, on this round, side A has won two points. The total score, after two rounds, is zero. (It is when the students compute the total score after each round that they perform addition of signed numbers without instruction.)

But there is more than adding signed numbers to learn from the matrix game. Here is a set of matrices, not only to provide variety but to test out the matrix itself. Suppose one side wins all the time? Is the matrix fair? (Notice: *not* is one *side* or the other *side* fair!) Thus the children become acquainted with bias and its causes and results. Depth as well as simplicity makes the Madison Project appeal to the teacher, too.

* The Madison Project is so named because it originated in The Madison School of Syracuse, New York. Its aim is to bring to the classroom all the best features of various studies for improving mathematics instruction. A number of schools are now using the materials developed at Madison.

ADDITIONAL MATRICES

I.
$$\begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c}1\\2\\3\end{array} & \begin{bmatrix} +1 & 0 & -1 \\ +2 & -1 & +1 \\ +3 & +3 & 0 \end{bmatrix} \end{array}$$

II.
$$\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} +3 & 0 & -3 \\ -2 & -1 & -1 \\ +3 & +3 & 0 \end{bmatrix} \end{array}$$

III.
$$\begin{array}{cc} \begin{array}{c}1\\2\\3\end{array} & \begin{bmatrix} +3 & 0 & -3 \\ -2 & -1 & 0 \\ +3 & +3 & 0 \end{bmatrix} \end{array}$$

IV.
$$\begin{bmatrix} +3 & 0 & -3 \\ -2 & +2 & 0 \\ -1 & -2 & +3 \end{bmatrix}$$

V.
$$\begin{array}{cc} \begin{array}{c}1\\2\\3\end{array} & \begin{bmatrix} +1 & -1 & 0 \\ +4 & -3 & -1 \\ -5 & +4 & +1 \end{bmatrix} \end{array}$$

VI.
$$\begin{bmatrix} +1 & -2 & +1 \\ 0 & +3 & -3 \\ -1 & -1 & +2 \end{bmatrix}$$

VII.
$$\begin{array}{cc} \begin{array}{c}1\\2\\3\end{array} & \begin{bmatrix} 1 & 2 & -3 \\ -3 & 2 & 1 \\ 2 & -4 & 2 \end{bmatrix} \end{array}$$

VIII.
$$\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 1 & 1 & 2 \\ 5 & -5 & 0 \\ -5 & 5 & 0 \end{bmatrix} \end{array}$$

IX.
$$\begin{bmatrix} -1 & 0 & -1 \\ 5 & -5 & 0 \\ -5 & 5 & 0 \end{bmatrix}$$

Are any matrices unfair, biased?

The Project material covers many topics besides matrices. Equations are pursued using a "balance" and a "missing link" or "pronumeral" technique. The children soon solve simultaneous equations with enjoyment. Words like "truth-set" and "solution set" enter the scene easily and naturally. Transform operations hold no terrors; identities resolve without confusion.

If you don't like the idea of bringing dice into the classroom, the Point-set game provides an even better way of graphing coordinates. It is a game of pure skill and it is very exciting. Everyone has fun locating points, but the strategy involved maintains long range interest for the most advanced. *Mirabile dictu*, all parts of the intellectual range learn to work together as a team.

If you are wondering whether or not the Madison Project would be successful with your group, you should know that it has been (and is being) tried with below normal, normal, and gifted children. The Madison Project "works" at all three levels. The more homogeneous the group, the better the results.

EDITOR'S NOTE. The concept of a simple negative number is not difficult. Many pupils in the primary grades have gained this rather well and some of them can combine positive and negative values when small numbers are involved. The "matrix game" seems very well suited as a medium and as abilities develop the numbers may be increased and even fractions can be introduced. Miss Lutz testifies that all levels of pupils in the seventh grade both enjoy and learn in this procedure. We need to experiment with many types of new materials but always we should keep in mind the contribution of these materials to the mathematical goals we want for the children. It would be easy to find new materials that are interesting, stimulating, and easily learned and yet we may want to reject them because they seem to serve no purpose except perhaps motivation and pleasure. However, we must not judge superficially because a program or device that on the surface may seem superficial may actually be building a rather fine understanding of mathematical relationships.

Standard Time

Vera Sanford
Oneonta, New York

"THE EARTH, YOU KNOW, turns round once in every 24 hours, or, in common language, the sun moves round the earth in that time; in what time, then, will the sun travel over 15 degrees? and why? over 1° (degree) of motion? over 1′ (minute) of motion? By the foregoing, we see that every degree of motion makes a difference in time of 4 minutes, and every minute of motion a difference of 4 seconds. Now, since longitude is reckoned in degrees round the earth, can you tell me how to find the difference in time between one place and another, after knowing their difference in longitude?"* When this book was written, the relation between the difference in longitude of two places and their difference in time was an academic matter. Travel and communications were slow. Local time was sufficient.

In popular interpretation, twelve o'clock noon was the time when the sun reached its highest point in the sky and when, in consequence, the shadow of a post was at its shortest for that day. In the northern hemisphere, this was the time when the sun was directly south of the observer: the time when the shadow of a window casing or of a door jamb would point due north. Accordingly, a "noon mark" would be scratched on a window sill or on the floor and noon would be the time when the edge of the shadow rested on the mark. Although this method ignored the equation of time, long known to astronomers, it was sufficient for ordinary purposes.

Certain writers of arithmetics made use of the relation between difference in sun time and difference in longitude in verbal problems. For example:

Supposing a meteor should appear so high that it could be seen at once by the inhabitants of Boston, 71°3′, of Washington, 77°43′, and of the Sandwich Islands, 155° west longitude; if the time be 47 minutes past 11 o'clock of Dec. 31, 1847, at Washington, what will be the time at Boston, and at the Sandwich Islands?

Daniel Adams, *Arithmetic*, Boston 1848, p. 172

Boston is situated about 6°40′ E. longitude from the city of Washington; when it is 2 o'clock at Washington, what o'clock is it at Boston?

I recollect of reading a story once of a gentleman going to a foreign country, who had a fancy to look at a bright star every evening, at the same moment, with a certain lady whom he left behind, and they agreed to look at 9 o'clock; but, it seems what, when the gentleman was in a different longitude, the time would of course be different; as, for instance, when he was in longitude differing 30° W. from where the lady was, she most probably had retired to rest, and was, perhaps, asleep, while he was gazing at the star. Can you tell me what o'clock it was, then, where she was?

Roswell C. Smith, *loc. cit.*, p. 232.

With the building of railroads, local time was no longer sufficient. In the 1870's, for example, trains between Hartford and New York left Hartford on New York time while those between Hartford and Boston ran on Boston time. The Hartford railroad station had clocks showing local time, New York time, and Boston time. A man arriving at Buffalo from Portland, Maine, at 12:15 Portland time, would find it was 11:40 by Buffalo time, but 12:00 by the New York Central clock and 11:25 by the Lake Shore clock. There were upwards of 500 railroads in the United States in the eighteen seventies and over 70 kinds of railroad time.

This situation did not affect the British railroads where extreme differences in longitude were not great and Greenwich time sufficed, but in the United States and in Canada, with differences in longitude as great as 60°, the situation was complicated. The problem was solved in the two countries at about the same time, with Sandford

* Roswell C. Smith, *Practical and Mental Arithmetic*, 1842, ed., pp. 231–2.

Fleming in Canada and Charles Ferdinand Dowd in the United States taking active parts.

Dr. Dowd (1825–1904), a Yale graduate, spent his adult life as a teacher and finally as principal of the Temple Grove Ladies Seminary at Saratoga Springs, New York, a school whose property was later acquired for Skidmore College. Dr. Dowd's work on a time system for North America was an altruistic project lasting for a dozen years.

His first idea was a single time system for the United States, but as this lost all connection between clock time and sun time, he discarded it in favor of time zones—four belts 15° of longitude in width. The meridian of Washington would have been an appropriate reference line for this, but as this line was 77°1′ west of Greenwich, Washington time differed from Greenwich time by 5 hr. 8 min. A change to make the center of the first time zone at the 75° meridian would make the difference in time just five hours. It should be noted that the Greenwich meridian had recently been adopted as the reference meridian for making maps replacing the use of the meridian of the capital city or of the chief observatory of a country.

In October, 1869, Dr. Dowd presented his plan of time zones reaching $7\frac{1}{2}$ degrees east and west of the 75°, 90°, 105°, 120° meridians to a convention of railway trunk lines in New York City. The association asked him to develop the system further.

To complete the system, it was necessary to chart the dividing lines of the time zones to cause the least inconvenience so far as state boundaries were concerned, and to list the correction factor for the conversion to *Standard Time* of the local time of each railroad station in the country.

The plan was approved by scientists at Yale and elsewhere. But railroad rate wars occupied the attention of subsequent railway conventions. Dr. Dowd was able, however, to gain the approval of the New England Railway Association and that of the Western and Southern Association but these did not press the change because "most passengers went only short distances."

In 1879 the American Society for the Advancement of Science took up the matter and urged that the railways adopt Standard Time. The railways appointed a Mr. William F. Allen to investigate the matter, and the change was made in 1883.

According to the *New York Times* of Nov. 19, 1883, "At nine o'clock yesterday morning, the pendulum of the standard clock in the Western Union Telegraph Building . . . was set at rest for 3 minutes and 58.38 seconds." Standard Time had come into existence.

There were the usual non-conformists. A man in Cleveland kept his clocks on local time to the annoyance of his family, and to their later delight in telling the tale. It was comparable to the situation when Daylight Saving Time was first instituted in World War I when certain people resisted the change on the score that they were satisfied with "God's time."

Thus the creation of Standard Time zones in the United States was due to an intolerable condition that like Topsy had "just growed," to the industry and perseverance of a school man, to the support given by the American Society for the Advancement to Science, and to the action of the Railway Association.

EDITOR'S NOTE: Many people will not realize that "Standard Time" as we know it is only seventy-five years old. The system of time belts and the corresponding hourly changes became necessary with the more rapid and more common transportation of goods and people. Measures were similarly standardized when the demands of trade made this necessary. With rapid rates of travel and with radio and television programs originating at great distances, people have become much more "time conscious." Certainly the arithmetical and the geographical factors in the study of time should be used cooperatively when this topic is being studied.

The editor is always amused when a newspaper reports that a wedding took place at "high noon" which implies that at this moment the sun was at its daily zenith. He has found but one clergyman who had given the topic serious thought. A good student in the upper grades might well explore such topics as "the equation of time" as an area for extra study.

Professor Sanford retired last summer from her position as chairman of the mathematics department at the State Teachers College at Oneonta.

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Compiled by JULIA ADKINS
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What Is a Number?—A Bulletin Board Display

There has been a tendency to use just one concept in teaching the meaning of a number to arithmetic students. This concept is the decimal system, which indicates that a number is composed of so many units of ones, tens, hundreds, etc.

One can illustrate this concept by using individual, and bundles of, sticks, toothpicks, straws, strips of paper, and other materials. The children can grasp this concept by handling these objects and putting the correct numbers of each in the proper columns. Thus, carrying and borrowing, as well as place value can be made visual.

But there are other ways of looking at a number. This year I organized a bulletin board which tried to show several ways of looking at a number. I called the bulletin board, "What Is A Number?"

Using the number 16, I indicated with headings that the number 16 might be thought of as denoting size, time, sequence, quality, and quantity.

Also, the number 16 was shown on a graduated number scale, which presented both positive and negative numbers. A number scale ought to make the relationship of one number to another in order more apparent.

The decimal system was illustrated in two ways—by a picture of an abacus, and with strips and bundles of strips in proper columns.

There is an idea that one might think of a number in terms of how many more numbers should be added to the original number to make the next multiple of ten. So, number 16 was illustrated with ten colored paper blocks in one column, and six blocks in another column with space for four more blocks indicated in different color.

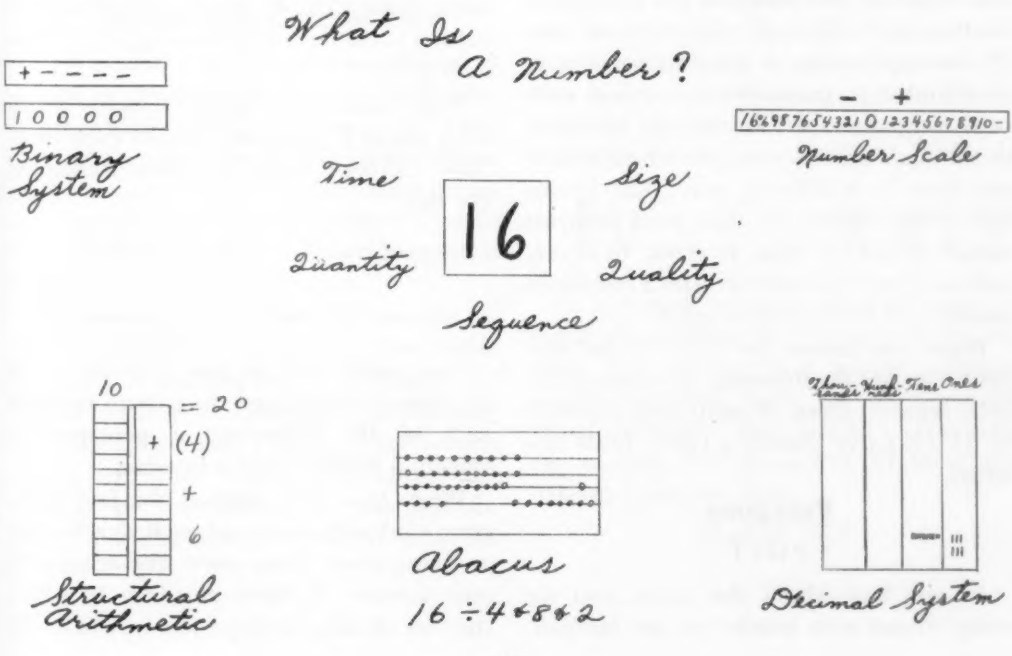
At the bottom of the bulletin board, the number 16 was shown as the multiple of the numbers 4, 2, and 8.

Finally, the idea that a number might be shown using just two symbols, instead of the regular nine digits and the zero, was indicated by illustrating the number 16, using the symbols 1 and 0 and + and −. This was how the bulletin board introduced the binary system to the students.

Red, yellow and blue colored construction paper gave the bulletin board sparkle and unity to the ideas presented.

Contributed by

LUCILE LAGANKE
Wilmington College, Ohio



Work-Type Lessons for Grade Six

In the field of mathematics there is less opportunity than in many other fields for students to read widely and to forge ahead as interest is aroused. Those who find arithmetic easy can work ahead in the book perhaps. However, it is to the students' advantage to have a new process presented by the instructor who can teach more meaningfully than can a textbook. The problem then is to supply material which will challenge the interest and ability of good students and also provide concepts of mathematical value in different types of situations.

To meet the need of a busy classroom teacher this material has been prepared. It is not intended for slow learning arithmetic students. It is prepared for the students who do not need so much practice on the regular assignments, but could be challenged to go deeper in the field of mathematics. It is prepared for students who have the ability to read and interpret directions without asking the teacher questions. It is prepared with the aim of giving such students: (1) practice in reading and following explicit directions; (2) the opportunity to work independently on a work-type project when required textbook assignments are completed; (3) additional mathematical concepts which will be met later in a different way. The lessons have been written so that good students should be able to read, to think, to figure, and sometimes to construct with a minimum amount of help from the teacher.

Before the lessons are given to the students, be sure the following items are available: scissors; sheets of cardboard about 8″ by 11″; a paper punch; a ruler; brass fasteners.

Polygons

Part I

As you look about the room, you see many shapes with which you are familiar, and which you can name: as a square, a rectangle, and a triangle. These figures, as well as all other figures which are enclosed by straight lines are called *polygons.* Today we are going to make some polygons.

Do you know what a pentagon is? Or a hexagon? Or an octagon? The dictionary will tell you the meaning of each one. When you are sure you know the meaning of each, you may begin your construction.

You are going to need a sheet of cardboard, a ruler, scissors, a paper punch, and brass fasteners.

Do you remember that it takes two dots to locate a straight line? Then measure carefully $\frac{1}{2}''$ from the narrowest side of the cardboard. Did you measure in two different places so that there will be two dots through which to draw the line? If so, then draw the line the entire width of the cardboard so that you have a strip $\frac{1}{2}''$ wide and about 8″ long. Cut off this strip and then, measuring carefully, cut it in half. You now have two strips about $\frac{1}{2}''$ by 4″. Repeat this process of cutting strips until you have eighteen strips each $\frac{1}{2}''$ by 4″.

Now, taking the paper punch, punch holes about $\frac{1}{2}''$ from each end of each strip. Your eighteen strips will look something like this:

Now comes the best part. You are to use the brass fasteners and these strips to make each of the following: a pentagon; a square; a triangle; and a hexagon.

When you have finished the four polygons, work with them and see if you discover anything about them worth discussing with your teacher. Is there any way in which they are all alike? Is there any way in which

one differs from the others? Write down, in complete sentences, anything you have discovered, and lay the paper on the teacher's desk. Be sure your name is on the paper.

PART II

The work we did in the last lesson should have led you to discover a very important bit of knowledge. This important fact is used over and over again in construction work. As you go about the neighborhood of downtown where there are large steel structures being built, look at the framework carefully and see if there is a certain kind of polygon used over and over. Keep this problem in mind until you have a chance to observe some real construction. You will be expected, before the end of the semester to write a paragraph telling what was important about the cardboard construction you did in the last lesson. You will want to include in that paragraph, the reason why that information is useful to men who build mighty structures.

Contributed by
VIOLET SHERWOOD
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A Device for Grade One

First graders can use the crimped covers of milk bottles to build with tens and ones. Let the children bring in the covers. There is more time for understanding if the covers do not come in too rapidly. The more colors and patterns on the covers (different products of different dairies) the more interesting is the game.

Suppose thirteen covers are brought in the first day. Ten are pinned in a row across the top of a bulletin board, and three are pinned on as the beginning of the second row. The children see 13 as 1 ten and 3 ones.

The next day eleven more covers are brought in and pinned on the bulletin board. The second row is finished and third row begun. Children see the 24 as 2 tens and 4 ones.

Continue till there are 10 rows of ten. Children can learn much from this game: Counting by 1's and by 10's; seeing the importance of 10 in our number system; gaining readiness for addition and subtraction.

Number Recognition in Kgn.

A large worktable is in the back of kindergarten or first grade room. On it are stamp pads and a number of stamps to make ponies, cowboys, trains, airplanes. . . . When a pupil has finished his work, he asks permission to work at the worktable. The teacher writes a number, say 7, on a small sheet of manila paper. It is the pupil's job to stamp exactly 7 ponies, or whatever he likes, on the sheet. He counts many times the figures he stamps and figures many times the number he still needs to make. There is a grieved look on teacher's face if the number he stamps is not correct; there is reward if it is correct. Teacher stamps the paper, if correct, as APPROVED with her initials, and he walks away as happy as if it were a college diploma. He takes it home to mother.

Not for the Romans

Claudia said she could take one from four and have five left. Then Julius showed her how to take eleven from twenty-nine and have twenty left. Here is how they did it. You can invent other tricks with Roman Numerals.

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In a remote region it was customary to name all children after characters in the bible. One minister was puzzled when a mother asked to have her child named "Pisalem Sieve." She couldn't remember the spelling but proceeded to show the minister from her bible

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In "The Good Old Days"*

Advertisement

SLATED ARITHMETICS

This consists of the application of an artificially slated surface to the inner cover of a book' with flap of the same opening outward, so that students may refer to the book and use the slate at one and the same time, and as though the slate were detached. When folded up, the slate preserves examples and memoranda till needed. The material used is as durable as the stone slate. The additional cost of books thus improved is trifling.

Prices

		Slated
Primary	$.25	$ .32
Intellectual	.40	.48
Practical	.90	1.00

* The above item was sent in by Professor Jesse Osborn of St. Louis. Prices were lower 100 years ago when slates were common.

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